Decentralized and scalable conflict resolution strategy for multi-agents systems

Lucia Pallottino, Vincenzo Giovanni Scordio, Emilio Frazzoli and Antonio Bicchi

***Abstract*— A decentralized cooperative collision avoidance control policy for planar vehicle recently proposed is herein considered. Given some simple conditions on initial configurations of agents, the policy is known to ensure safety (i.e., collision avoidance) for an arbitrarily large number of vehicles. The method is highly scalable, and effective solutions can be obtained for several tens of autonomous agents. On the other hand, the liveness property of the policy, i.e. the capability of negotiating a solution in finite time, is not yet completely understood. First a 3D workspace extension is proposed. Furthermore, based on a condition on targets configuration previously proposed, some general results on the liveness property are reported. Finally, qualitative evaluations on the strategy and on the proposed target sparsity condition are pointed out.**

***Keywords*— Multi-vehicle systems, traffic management, decentralized control**

I. INTRODUCTION

In the last few years, the problem of safely coordinating the motion of several agents sharing the same environment has received a great deal of attention, both in robotics and in other application domains. Decentralized control policies, based on locally available information, are scalable to large-scale systems, and robust with respect to single-point failures. Few decentralized algorithms have appeared recently, e.g. [4], [5] for holonomic robots, and [6] for aircraft-like vehicles. The literature on flocking and formation flight, which has flourished recently (e.g., [7]–[9]), while ultimately leading to conflict-free collective motion, does not address individual objectives, and agents are not guaranteed to reach a pre-assigned individual destination. Recently, Kyriakopoulos and coworkers introduced decentralized control policies ensuring the safe coordination of several non-holonomic vehicles [10]. A number of techniques have been developed for omni-directional (holonomic) robots, most of them requiring some form of central authority, either prioritizing robots off-line, or providing an online conflict-resolution mechanism, e.g., [1]–[3].

A safe and decentralized collision-free strategy has been recently proposed in [11] for mobile agents evolving on the plane. Agents are modeled as nonholonomic vehicles, constrained to move at constant speed and with bounds on the curvature. The environment in which the agents move is considered to be unbounded and free of obstacles. Furthermore, the information available to each agent is the position and orientation of nearby agents, within a certain sensing or communication radius. In particular, agents are not supposed to communicate explicitly their goals or their velocities. All agents make decisions based on a common set of rules which are decided a priori, and rely on the assumption that other agents apply the same rules. Some areas of application of the considered problem include air traffic control, manufacturing plants, automated factories, and intelligent transportation systems.

The control policy, introduced in [11], is spatially decentralized, and highly scalable. A first contribution of the paper is the extension of the proposed decentralized strategy to the 3D workspace case by introducing a altitude-layer structure. The application to the airtraffic management system problem is straightforward.

The proposed strategy was proven to be safe for an arbitrarily large number of agents, [11]. Through a large number of simulation the strategy has showed to be very effective in negotiating conflicts of several tens of agents. However, its liveness property is not yet completely understood. In other words, while it is known that the proposed policy never causes collisions under some mild assumptions on the initial conditions, it is not clear under what conditions on the initial and final configurations the policy ensures that each vehicle will reach the intended destination in finite time.

Simple conditions involving only final configurations that can be sufficient to exclude livelocks, hence guarantee a solution to be found in finite time have been proposed. Unfortunately, the formal verification of the conjecture appears to be overwhelmingly complex. Some preliminary results have been obtained in [12] with a probabilistic approach based on classical Monte Carlo methods [13]. In that work, we assessed the correctness of the conjecture in probability through the analysis of the results of a large number of randomized experiments for systems with 10 agents and a fixed dimension. In this paper, several extensions to the results obtained in [12] are provided. In particular, we assert that the the conjecture is correct in probability for an entire class of problems for which the number of agents has value in $\{2, \ldots, 10\}$ and the safety disc radius assumes different dimensions.

The proposed sparsity condition involving final configuration is far from being necessary for liveness. In

L. Pallottino, V. G. Scordio and A. Bicchi are with Interdepartmental Research Center "E. Piaggio", University of Pisa, Italy

E. Frazzoli is with University of California at Los Angeles, Los Angeles, CA

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other words, in many cases that do not satisfy the targets condition all agents will eventually reach their goal. Another contribution reported in this paper is the qualitative evaluation of the conflict resolution policy and of the proposed target sparsity condition.

The paper is organized as follows: in section II basic tools of probability estimation are described. In section III we introduce some notation, define the problem we wish to address and briefly report the generalized roundabout policy. In section IV the liveness property of the policy is analyzed and the *sparsity condition* is described. In section V the probabilistic approach is applied to the liveness verification problem and new results are reported. Finally, in section VI, we draw some conclusions and discuss some directions for future work.

II. The probabilistic approach

We report here a brief account on the basic tools of probability estimation described in [12]. The reader is invited to refer to the specialized literature (e.g. the excellent book [13]) for more details.

Given a dynamical system D subject to uncertainties Δ and a predicate P_D defined on D which we want to verify. Let $\mathcal{B}$ be the bounded set in which uncertainties are confined and $f_\Delta(\Delta)$ the associated probability density function. Probabilistic verification consists in evaluating, with a prescribed confidence, the probability

$$p_D := \mathrm{PR}_{\mathbf{\Delta}}\{P_D(\mathbf{\Delta})\} = \int_{\mathcal{G}} f_{\mathbf{\Delta}}(\Delta) d\Delta,$$

where $\mathcal{G} \subseteq \mathcal{B}$ denotes the *good set* of $\Delta \in \mathcal{B}$ for which $P_D(\Delta) = true$.

Given a *performance function* $J_D(\mathbf{\Delta})$ of system D, the probability that a given performance level γ is attained under uncertainties as above can be expressed by the predicate $P_D(\mathbf{\Delta}) = \{J_D(\Delta) \leq \gamma\}$.

The measure of the predicate veridicity is given by the volume ratio $r = Vol(\mathcal{G})/Vol(\mathcal{B})$ that can be evaluated by a Monte Carlo approach if a uniform distribution function on $\mathcal{B}_d$ is considered. Indeed, under this assumpiton $p_D = r$. Let us denote by $\mathbf{\Delta}^i$, $i = 1, \ldots, N$ N random samples within $\mathcal{B}$. An estimate of r based on the empirical outcomes of the N instances of the problem is given by $\hat{p}_D(N) = \frac{1}{N}\sum_{i=1}^{N} I_{\mathcal{G}}(\mathbf{\Delta}^i)$ where $I_{\mathcal{G}}(\mathbf{\Delta}^i) = 1$ if $\mathbf{\Delta}^i \in \mathcal{G}$ and 0 otherwise.

This result provides a finite N such that the empirical mean $\hat{p}_D(N)$ differs from the true probability p_D less than ϵ with probability greater than $1 - \delta$, i.e. $Pr\{|p_D - \hat{p}_D(N)| < \epsilon\} > 1 - \delta$, for $0 < \epsilon, \delta < 1$. To determine the minumum number N the Chernoff bound [14] can be used:

$$N > \frac{1}{2\epsilon^2} \log\left(\frac{2}{\delta}\right). \tag{1}$$

Notice that the sample size N, given by (1), is independent of the size of $\mathcal{B}$ and of the distribution $f_{\mathbf{\Delta}}(\Delta)$.

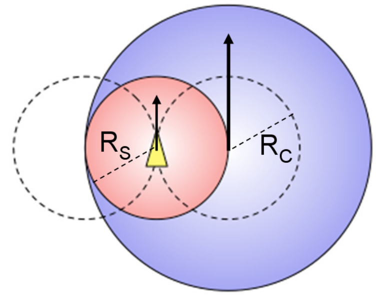

Fig. 1. The reserved disc of a nonholonomic vehicle with bounded angular velocity.

III. Problem Formulation and Coordination Policy

Let us consider n mobile agents moving on the plane at constant speed, along paths with bounded curvature. Let the configuration of the i-th agent be specified by the triple $g_i = (x_i, y_i, \theta_i)$, where x_i and y_i specify the coordinates of a reference point on the agent's body with respect to an orthogonal fixed reference frame, and the heading θ_i is the angle formed by a longitudinal axis on the agent's body with the $y = 0$ axis.

Each agent enters the environment at the initial configuration $g_i(0) = g_{0,i}$, and is assigned a target configuration $g_{\mathrm{f},i}$. The agents move along a continuous path according to the model

$$\begin{array}{rcl} \dot{x}_i(t) & = & v_i \cos(\theta_i(t)) \\ \dot{y}_i(t) & = & v_i \sin(\theta_i(t)) \\ \dot{\theta}_i(t) & = & \omega_i(t) \end{array} \tag{2}$$

where $\omega_i : \mathbb{R} \to [-\frac{1}{R_C}, \frac{1}{R_C}]$ is a bounded signed curvature control signal. Without loss of generality we can scale the control $\omega_i \in [-1, 1]$ by considering $R_C = 1$. Linear velocity v_i is constant and can be supposed equal to 1 for each agent without lost of generality.

A *conflict* is said to occur between two agents, whenever the agents become closer than a specified safety Euclidean distance d_{s}. Hence, associating to each agent a *safety disc* of radius $R_S = \frac{d_{\mathrm{s}}}{2}$ centered in the agent position a conflict occurs whenever two safety discs overlap.

A dynamic feedback control policy π is a map that associates to an individual agent a control input, based on a set of locally-available *internal variables*, and on the current configuration of other agents in the environment. The policy π is said *spatially decentralized* if it is a function only of the configurations of agents that are within a given alert distance d_{a} from the computing agent.

A brief description of the spatially decentralized policy is now reported for reader convenience. Further details can be found in [11] and [12].

Reserved disc: The proposed policy is based on the concept of *reserved disc*, over which each active agent claims exclusive ownership. Given the agent configuration g, the associated reserved disc has radius $R = 1 + R_S$, is centered in $(x^c, y^c) = (x + \sin(\theta), y - \cos(\theta))$ and inherits the

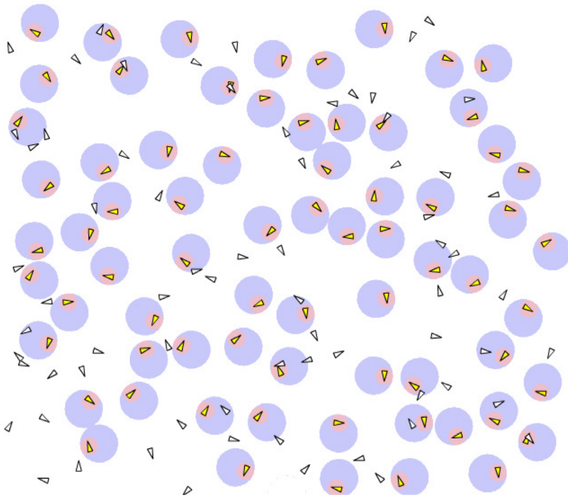

Fig. 2. A conflict resolution problem with 70 agents in narrow space, for which the proposed policy provides a correct solution. Initial configurations are identified by the presence of gray circles, indicating their reserved discs.

agent's heading θ, refer to Figure 1. Let $g^c = (x^c, y^c, \theta^c)$ be the configuration of the reserved disc, its dynamics is described by $\dot{g}^c = ((1+\omega)\cos\theta^c, (1+\omega)\sin\theta^c, \omega)$.

Notice that when the agent has control $\omega_i = -1$, corresponding to a maximum curvature radius clockwise turn, the center of the associated reserved disc is fixed, see Figure 1. Hence the reserved disk can be stopped at any time, by setting $\omega = -1$ and it can be moved in any direction, provided one waits long enough for the heading θ to reach the appropriate value.

The proposed *Generalized Roundabout Policy* is based on following 4 maneuvers; the reader can refer to [11] and [12] for a more detailed description.

Straight: associated to the control $\omega = 0$, steers the center of the agent's reserved disk towards the position it would assume at the target configuration;

Hold: as previously mentioned, setting $\omega = -1$, causes an immediate stop of an agent's reserved disk's motion.

Roll: if the path of the reserved disk is blocked by another stationary reserved disc, a possible action is represented by rolling in the counterclockwise direction on the boundary of the blocking disc. This action is performed without violating the safety constraints by setting $\omega = (1+R_S)^{-1}$.

Roll2: in general, the reserved disk of an agent will not necessarily remain stationary while an agent is rolling on it. In this case the contact can be lost and the rolling agent switches to this state, corresponding to $\omega = 1$. In this way the agent attempts to recover contact with the former neighbour, and to exploit the maximum turn rate when possible.

IV. Liveness Analysis

The policy described in the previous section provides effective solutions for large-scale problems, such as e.g. the 70-agents conflict resolution illustrated in fig. 2. Moreover, the policy was shown in [11] to be well-posed and safe. We are concerned here with the following property:

Liveness: The closed-loop hybrid system $\mathcal{S}_{\mathrm{GR}}$ is *live-lock free* if all agents reach their final destinations:

$$\forall i \in \{1, \ldots, n\}\, \exists t_{i\mathrm{f}} \geq 0 : \, g_i(t_{i\mathrm{f}}) = g_{\mathrm{f},i}. \tag{3}$$

Although in [11] the condition on targets location was proved to be sufficient for liveness in the case of two agents only, a general condition ensuring liveness for an arbitrary number of agents is not yet known.

Furthermore, in [12] we have reported two cases of livelock occurring for problems of four and n agents respectively. The analysis of these examples allows us to observe that livelock generation appears to be possible in cases where a number n of target configurations are closely clustered. Such observations lead us to define the following **Sparsity condition**: for all $(x, y) \in \mathbb{R}^2$ and for $m = 2, \ldots, n$,

$$\mathrm{card}\{(x_c, y_c) \in \mathcal{B}_f : \|(x_c, y_c) - (x, y)\|_2 < \rho(m)\} < m, \tag{4}$$

where $\mathcal{B}_f$ is the set of centers of reserved discs at the target, n is the number of agents and

$$\rho(m) = \begin{cases} (1+\cot(\frac{\pi}{m}))(1+R_S) & \text{for } m \geq 4, \\ 2(1+R_S) & \text{otherwise.} \end{cases} \tag{5}$$

In other words, any circle of radius $\rho(m)$, with $1 < m \leq n$, can contain at most $m-1$ reserved disc centers of targets.

We conjecture that the sparsity condition (4) is sufficient for liveness. As already mentioned, the conjecture is very difficult to prove analitically for $n > 2$. The probabilistic approach reported in II is applied in the following section to provide an estimate of the sufficiency of the sparsity condition.

A. Extension to 3D workspace

The sparsity condition allows a dynamic management of the agents. At each instant, it is possible to introduce agents in the system provided that the associated reserved discs do not overlap other reserved discs and satisfy the sparsity condition. In our approach we consider an agent that has reached the target configuration as not part of the system anymore, i.e. whenever an agent reach the target the number of agents in the system decrease of one. If in a 3D workspaces an altitude–layered structure is introduced the roundabout strategy can be easily extened. To each agent we now associate a *safety cylinder* whose base is the safety disc and height the same height of the layers. Conflicts need only to be resolved among agents moving within the same layer. Each agent can change its layer only if the sparsity condition is verified in the target layer and the reserved disc of the agent do not overlap any other reserved disc of agents in the target layer. This extension of the decentralized strategy has a straightforward application to the conflict resolution problems in the air traffic management system (ATMS).

V. Probabilistic verification of liveness

In order to apply the probabilistic verification approach described in section II, we now introduce some basic notation, the reader can refer to [12] for more details. Denote with $g_{0,i}$ and $g_{f,i}$ the initial and final configurations of agent i, respectively, and with $g^c_{0,i}$ and $g^c_{f,i}$ the configurations of the center of the reserved disc associated to $g_{0,i}$ and $g_{f,i}$, respectively. Furthermore, let $g^c_0 = \{g^c_{0,i}, i = 1, \ldots, N\} \in (\mathbb{R}^2 \times S^1)^N$ and $g^c_f = \{g^c_{f,i}, i = 1, \ldots, N\} \in (\mathbb{R}^2 \times S^1)^N$, where N is the maximum number of agents considered.

Consider a predicate $P_{\mathcal{S}_{GR}}(g^c_0, g^c_f)$, which is true if the dynamical system $\mathcal{S}_{GR}$ defined by the generalized roundabout policy provides a solution in finite time for initial and final configurations g^c_0 and g^c_f, respectively. We will restrict to uniformly distributed uncertainties $\Delta = (g^c_0, g^c_f) \in \mathcal{B}$, where $\mathcal{B} = \mathcal{B}_0 \times \mathcal{B}_f$, $\mathcal{B}_0 = \mathcal{B}_f = ([0, 800] \times [0, 700] \times [0, 2\pi))^N$. Accordingly, $\mathcal{G} \subset \mathcal{B}$ denotes the "good" set of problem data for which the predicate applies.

Let $\mathcal{C} \subset \mathcal{B}$ denote the set fulfilling the (4). Notice that the sparsity condition (4) is defined on the target configurations g^c_f only, hence $\mathcal{C}$ is a cylinder with axis $\mathcal{B}_0$. Verification of the conjecture above is tantamount to showing that

$$r = \frac{Vol(\mathcal{G} \cap \mathcal{C})}{Vol(\mathcal{C})} = 1.$$

In order to obtain an empirical estimate of r thorugh execution of numerical experiments, the predicate is modified in the form

$$P'_{\mathcal{S}_{GR}}(g^c_0, g^c_f) = \{J(g^c_0, g^c_f) \leq \gamma\},$$

where $J(g^c_0, g^c_f)$ denotes the time employed by the last agent to reach its goal, and γ is a threshold to be suitably fixed.

Finally, estimates of the ratio r have been evaluated by the probabilistic approach previously described. Preliminary results have been already reported in [12]. In that paper a fixed number of agents and a fixed value of the safety disc radius have been considered ($n = 10$ and $R_S = 18$).

Some extensions to those preliminary results are herein reported. In order to have accuracy $\epsilon = 0.0075$ with 99.25% confidence ($\delta = 0.0075$), it was necessary by (1) to run 50000 experiments, with initial and final conditions uniformly distributed in the configuration space $\mathcal{C}$. Samples were generated by a rejection method applied to uniform samples generated in $\mathcal{B}_n = ([0, 800] \times [0, 700] \times [0, 2\pi))^n \subset \mathcal{B}$, with $n \in \{2, \ldots, 10\}$ and $R_S \in \{2, \ldots, 18\}$. None of these 50.000 experiments failed to find a solution within time $\gamma = 4000$, hence $\hat{p}_D(N) = 1$. Hence, with 99.25% of confidence we can state that the sparsity condition is sufficient to guarantee liveness of the generalized roundabout policy to within an approximation of 0.75% for systems with a number of agents varying from 2 to 10 and safety radius from 2 to 18.

A. Qualitative evaluation of the sparsity condition and of the liveness of the policy

We are now interested in providing qualitative evaluations on sparsity condition on the targets and the liveness of the chosen policy .

The dimension of $\mathcal{C}$ in $\mathcal{B}$ depends on the value of the number of agents n and the value of the associated safety radius R_S. Figure 3 represents the normalized dimension of $\mathcal{C}$ in $\mathcal{B}_n$ with respect to variation of $n \in \{2, \ldots, 20\}$ and $R_S \in \{2, \ldots, 40\}$. In figure 4 the z-axis view is reported. Projections of the isodimensional curves on the (n, R_S) plane appear to be hyperbolas, i.e. $n\, R_S = const.$.

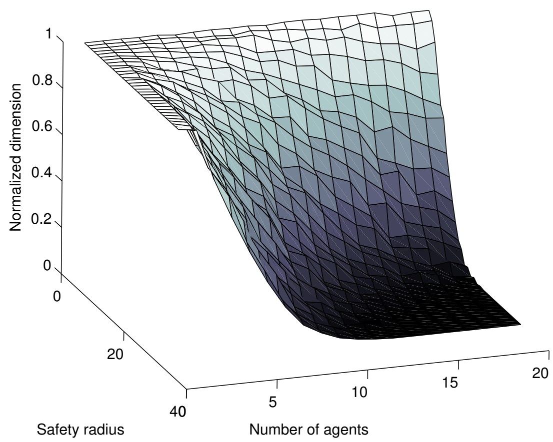

Fig. 3. The normalized dimension of $\mathcal{C}$ in $\mathcal{B}$ with respect to variation of n and R_S.

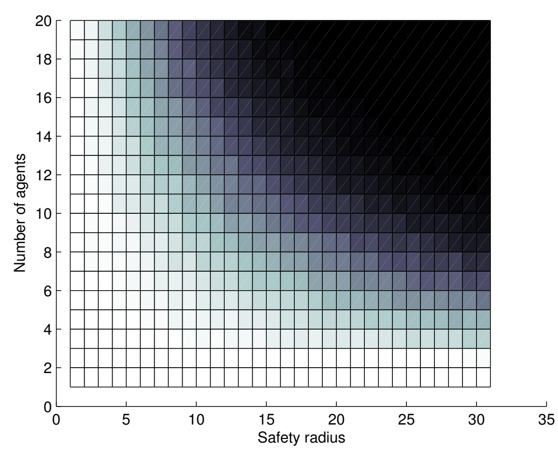

Fig. 4. Projections of the isodimensional curves on the (n, R_S) plane appear to be hyperbolas.

Using values of n and R_S such that the dimension of $\mathcal{C}$ in $\mathcal{B}_n$ is larger or equal to 95% we have verified, with the proposed probabilistic approach, that with 99% confidence the sparsity condition is sufficient to guarantee liveness of the generalized roundabout policy to within an approximation of 1%. For the remaining 5% of $\mathcal{B}_n \setminus \mathcal{C}$ more than 20000 simulations have been run. In the 96.433% of cases such simulations have terminated with the reaching of the goal configurations, i.e. no livelock has occurred. In conclusion, regarding the liveness property of the proposed Roundabout policy, we can affirm that for some particular values of n and R_S in more of $0.99 \cdot 0.95 + 0.96433 \cdot 0.05 = 99.8\%$ of cases all agents will eventually reach the goal configurations.

Furthermore, notice that for those value of n and R_S, the total space occupied by agents is around the $4-5\%$ of the whole workspace. To give an idea, in terms of agents occupancy this means that in a workspace of dimension 7meter $\times$ 8meter we are able to manage safely 10 agents with a safety disc diamater of 60 centimeters.

VI. Conclusions and future work

In this paper, we have considered a decentralized cooperative control policy for conflict resolution for multiple nonholonomic vehicles. The liveness properties of the policy has been investigated for an entire class of multi-agent systems. A conjecture on the final vehicle configurations, providing a sufficient condition for liveness, has been studied with a probabilistic method. Several extensions to previously obtained results are reported.

Future developments of this research will address tighter necessary and sufficient conditions for the generalized roundabout policy to apply, also with respect to the scale of the environment and the number of agents involved.

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