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A Robust Iterative Learning Control for Continuous-Time Nonlinear Systems with Disturbances

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ABSTRACT In this paper, we study the trajectory tracking problem using iterative learning control for continuous-time nonlinear systems with a generic fixed relative degree in the presence of disturbances. This class of controllers iteratively refine the control input relying on the tracking error of the previous trials and some properly tuned learning gains. Sufficient conditions on these gains guarantee the monotonic convergence of the iterative process. However, the choice of the gains is heuristically hand-tuned given an approximated system model and no information on the disturbances. Thus, in the cases of inaccurate knowledge of the model or iteration-varying measurement errors, external disturbances, and delays, the convergence condition is unlikely to be verified at every iteration. To overcome this issue, we propose a robust convergence condition, which ensures the applicability of the pure feedforward control even if other classical conditions are not fulfilled for some trials due to the presence of disturbances. Furthermore, we quantify the upper bound of the nonrepetitive disturbance that the iterative algorithm is able to handle. Finally, we validate the convergence condition simulating the dynamics of a two degrees of freedom underactuated arm with elastic joints, where one is active, and the other is passive, and a Franka Emika Panda manipulator.

INDEX TERMS Iterative Learning Control, Nonlinear control systems, Robustness, Robots

I. INTRODUCTION

STARTING from the 80s, a new control framework, namely iterative learning control (ILC), was introduced [1], [2]. The basic idea is to polish, iteratively, the current control input until the system is able to effectively perform the desired task. The iterative algorithm does not require any accurate description of the model, leading to good tracking performance without any substantial modification of the system dynamics, while incorporating persistent disturbances, e.g., gravity acceleration. Not surprisingly, ILC proved to be an excellent tool for repetitive tasks. Indeed, its field of applications are multiple, e.g., robotic manipulation [3], the wafer stage [4], manufacturing process [5], quadrotors [6], and soft robotics [7]–[10].

The iterative algorithm can be robust to disturbances [11] and can follow a switching policy between learning gains [12]. Additionally, the control law can involve a rectifying action for the initial state [13], can be combined with feedback control, e.g., proportional [14] or model predictive control [15], and can learn the desired trajectory even in the case of variations in the learning process [16].

The main problem when dealing with iterative processes is guaranteeing convergence. For continuous-time linear systems and discrete-time systems, it is possible to draw sufficient and necessary convergence conditions [17]–[19], while it is still an open problem for continuous-time nonlinear systems [19].

Disturbances such as model uncertainties [11], [20], error in the measurements, dynamic/external interactions [7], and actuation delays or faults [21] may cause a failure of the convergence condition.

Feedback controllers can mitigate the undesired effect of disturbances through the application of suitable high gains [22]. This leads to a profound alteration of the system dynamics, which is not acceptable in some applications like, for example, soft robotics [7]. In this case, the use of feedback control actions is strongly limit [23], and pure feedforward control action, e.g., ILC, is preferable. However, feedforward methods lack robustness in the case of disturbances. Thus, what happens in the case of disturbances? Which kind of disturbances can an iterative learning controller manage? What can we guarantee in terms of convergence?

The robustness of a pure feedforward iterative control law problem has already been widely investigated in the case of discrete-time systems [18], [20], [24], [25]. However, it is still under-studied for continuous-time systems. In [26], the sampled-data ILC algorithm for continuous-time systems can manage the time nonrepetitive disturbances, while in [27], the Authors tackle the same problem in the case of systems with a fixed relative degree equal to one, constant linear input and output fields, and saturated inputs.

In this paper, we design an iterative pure feedforward controller for multiple-input multiple-output (MIMO) continuous-time nonlinear systems with a generic fixed relative degree. We prove its convergence in the case of a great variety of disturbances. We distinguish disturbances on their dependency on the state or on time. Additionally, we classify them as repetitive or nonrepetitive depending on their occurrences w.r.t. the iteration domain. In [11], [18], [24], [25], the Authors already guarantee a bounded error in the presence of time-dependent nonrepetitive disturbance. We propose and prove a convergence condition (D-condition), which guarantees a robust convergence also in the presence of state-dependent nonrepetitive disturbances. Theoretically, we propose a necessary and sufficient converge condition for a restricted class of nonlinear systems. Then, we quantify the iteration-frequency and module of the nonrepetitive disturbances that the iterative algorithm can handle. Additionally, we prove that the D-condition does not modify the already known convergence results in [11], [18], [24], [25] dealing with time-dependent nonrepetitive disturbances.

Finally, we validate the D-condition on two simulated robotic systems varying disturbances types and output functions. The first robot is an underactuated compliant arm with two degrees of freedom (DoFs), in which the first elastic joint is active, while the other is passive. The second system is a *Franka Emika Panda* manipulator.

Notation

Let $I_m \in \mathbb{R}^{m\times m}$ be the identity matrix and $0_{n\times m} \in \mathbb{R}^{n\times m}$ be a zeros matrix. Let $f(\cdot), g(\cdot): x\in\mathbb{R}^n \to \mathbb{R}^n$ be two vector fields, $L_f g(x)$ stands for the Lie derivative of $g(x)$ along $f(x)$, i.e., $L_f g(x) = \frac{\partial g(x)}{\partial x} f(x)$. For any vector $v \in \mathbb{R}^n$, for any matrix $A \in \mathbb{R}^{n\times m}$, we denote with $||v||$ and $||A||$ their infinity norm. Let λ be a positive constant, for any vector $v\in\mathbb{R}^n$, we denote with $||v||_\lambda$ its $\lambda-$norm, i.e., $||v||_\lambda \triangleq \sup_t \left\{||v||\mathrm{e}^{-\lambda t}\right\}$. Let $y: t\in\mathbb{R}\to\mathbb{R}^n$ be a vector function, we denote with $y^{(i)}(t)$ its $i-$th time derivative. Let U be a set, we use the notation $\#U$ to indicate its cardinality. Finally, all physical units may be assumed to be expressed in SI (MKS), and angles in radian.

II. PROBLEM DEFINITION

Let us consider an iterative process, where $j\in U$ is the iteration index, and U is the iteration set. The class of continuous-time nonlinear systems under analysis can be written as

$$\begin{cases} \dot{x}_j(t) = f_{\mathrm{n}}(x_j(t)) + g(x_j(t))u_j(t) \\ \qquad + d^{\mathrm{px}}(x_j(t)) + d^{\mathrm{pt}}(t) + d_j^{\mathrm{rx}}(x_j(t)) + d_j^{\mathrm{rt}}(t) & (1) \\ y_j(t) = h(x_j(t)) + d_j^{\mathrm{rty}}(t)\,, & (2) \end{cases}$$

with $x_j(0)$ as initial condition, $x_j(t)\in\mathbb{R}^n$ is the state vector, $t\in[0,t_{\mathrm{f}}]$ is the time variable, t_{f} is the terminal time, $u_j(t)\in\mathbb{R}^m$ is the control action, $y_j(t)\in\mathbb{R}^{n_{\mathrm{y}}}$ is the output, $h(\cdot):\mathbb{R}^n\times[0,t_{\mathrm{f}}]\to\mathbb{R}^{n_{\mathrm{y}}}$ is the output map, $f_{\mathrm{n}}(\cdot):\mathbb{R}^n\times[0,t_{\mathrm{f}}]\to\mathbb{R}^n$ and $g(\cdot):\mathbb{R}^n\times[0,t_{\mathrm{f}}]\to\mathbb{R}^{n\times m}$ are the drift and control vector field, respectively. Additionally, the system is affected by disturbances $d^{\mathrm{px}}(x_j(t))$, $d^{\mathrm{pt}}(t)$, $d_j^{\mathrm{rx}}(x_j(t))$, $d_j^{\mathrm{rt}}(t)\in\mathbb{R}^n$, and $d_j^{\mathrm{rty}}(t)\in\mathbb{R}^{n_{\mathrm{y}}}$ which we classify in relation to their dependency on iteration, state and time domain. In particular, considering the iteration domain $j\in U$, we distinguish between repetitive and nonrepetitive disturbances. Furthermore, we divide them into state disturbances and time disturbances, respectively. It is instrumental for the development of the method to introduce the following definitions.

Definition 1. *A disturbance $d^{\mathrm{px}}(\cdot): U\times[0,t_{\mathrm{f}}]\times\mathbb{R}^n\to\mathbb{R}^n$, Lipschitz, and bounded is said to be state-repetitive (or state-persistent).*

Definition 2. *A disturbance $d^{\mathrm{pt}}(\cdot): U\times[0,t_{\mathrm{f}}]\to\mathbb{R}^n$, Lipschitz, and bounded is said to be time-repetitive (or time-persistent).*

Definition 3. *A disturbance $d_j^{\mathrm{rx}}(\cdot): T\subset U\times[0,t_{\mathrm{f}}]\times\mathbb{R}^n\to\mathbb{R}^n$, Lipschitz and bounded, i.e., $\max_t\left|\left|d_j^{\mathrm{rx}}(x_j(t))\right|\right| = \bar{d}_j^{\mathrm{rx}}$ is said to be state-nonrepetitive.*

Definition 4. *A disturbance $d_j^{\mathrm{rt}}(\cdot): U\times[0,t_{\mathrm{f}}]\to\mathbb{R}^n$ ($d_j^{\mathrm{rty}}(\cdot): U\times[0,t_{\mathrm{f}}]\to\mathbb{R}^{n_{\mathrm{y}}}$) bounded, i.e., $\max_t\left|\left|d_j^{\mathrm{rt}}(t)\right|\right| = \bar{d}_j^{\mathrm{rt}}$ ($\max_t\left|\left|d_j^{\mathrm{rty}}(t)\right|\right| = \bar{d}_j^{\mathrm{rty}}$) and such that $d_j^{\mathrm{rt}}(\cdot)\neq d_i^{\mathrm{rt}}(\cdot)$ ($d_j^{\mathrm{rty}}(\cdot)\neq d_i^{\mathrm{rty}}(\cdot)$), $\forall i\neq j\in U$ is said to be time-nonrepetitive.*

It is worth highlighting that, nonrepetitive disturbances, i.e., Def. 4, have already been widely studied in the literature, e.g., [11], [20], [26], [28]. However, the other types of disturbances have not been properly analyzed yet. In the following remark, we present a few practical examples of these definitions.

Remark 1. *State-repetitive disturbances (Def. 1) can represent an external force field, e.g., an unmodeled gravity vector* *in the dynamics of a robot. Time-repetitive disturbances (Def. 2) can model additive uncertainties in the system nominal model. It is worth remarking that repetitive disturbances are present at each iteration of the whole iterative process.*

State-nonrepetitive disturbances (Def. 3) can derive from the interaction between a robot and the environment or actuators failure/delays. Time-nonrepetitive disturbances (Def. 4) are disturbances with no relation with the state, e.g., measurements noise. It is worth remarking that nonrepetitive disturbances occur only for a few iterations (or change at each iteration) during the whole learning process.

Assumptions

We assume for the system (1)-(2) what follows:

A1) the system (1)-(2) is square, i.e., $n_y = m$.

A2) The system (1)-(2) has a fixed relative degree (vector) r_v such as $r_v = [r_1, \dots, r_m]$ (see, e.g., [29]):
- $L_{g_z} L^s_{f_n} h_i(x) = 0,\ i, z \in [1, m],\ s \in [0, r_i - 1]$.
- $\text{rank}\{D(x)\} = m, \forall x \in \mathbb{R}^n$, such as $D(x) \in \mathbb{R}^{m \times m}$ and $D_{ij}(x) = L_{g_i} L^{r_j - 1}_{f_n} h_j(x)$ with $i, j \in [1, m]$.

The matrix $D(x)$ is called the decoupling matrix. Additionally, we assume that $r_1 = \cdots = r_m = r \in \mathbb{N}$.

A3) The initial condition $x_j(0) \in \mathbb{R}^n$ is such that $x_j(0) = x_d(0), \forall j \in U$.

A4) $f_n(\cdot), g(\cdot), h(\cdot), L^s_f h(\cdot), s = 1, \cdots, r$, and $D(\cdot)$ are globally Lipschitz with constants $\bar{f}, \bar{g}, \bar{h}, \bar{\Phi}_s$, and $\bar{\eta} \in \mathbb{R}^+$, respectively, i.e.,$||f_n(\hat{x}) - f_n(\bar{x})|| \leq \bar{f}\, ||\hat{x} - \bar{x}||,\ \hat{x}, \bar{x} \in \mathbb{R}^n$.

A5) The desired output trajectory $y_d : [0, t_f] \to \mathbb{R}^m$ is feasible, continuous and differentiable for r times, $\forall t \in [0, t_f]$.

It is worth noting that, thanks to assumption A5), there exist bounded u_d, x_d, and y_d, which are the desired control input, state and output, respectively[1], such that $\dot{x}_d(t) = f_n(x_d(t)) + g(x_d(t)) u_d(t)$ and $y_d(t) = h(x_d(t))$.

Goals

Considering the disturbed system (1)-(2), given the desired trajectory $y_d(t) : [0, t_f] \to \mathbb{R}^m$, and assumptions A1)-A5). The main purpose of this paper is to investigate the robustness of the iterative feedforward control law $u_j(t)$ in the presence of disturbances $d^{px}(x_j(t))$, $d^{pt}(t)$, $d^{rx}_j(x_j(t))$, $d^{rt}_j(t)$, and $d^{rty}_j(t)$. In particular, we summarize the goals of this work as follows.

G1) Design an iterative feedforward control law $u(t) : [0, t_f] \to \mathbb{R}^m$ able to follow $y_d(t)\ \forall t \in [0, t_f]$, i.e., $\lim_{j \to +\infty} ||y_d(t) - y_j(t)||_\lambda = 0$.

G2) Propose a robust convergence condition, namely D-condition, which guarantees G1) even in the presence of state-nonrepetitive disturbances $d^{rx}_j(x_j(t))$.

G3) Find an upper-bound of the state-nonrepetitive disturbances $d^{rx}_j(x_j(t))$, which can be dealt with by the convergence condition proposed in G2).

G4) Prove that the D-condition in G2) handles the presence of time-repetitive $d^{pt}_j(t)$ and time-nonrepetitive $d^{rt}_j(t)$ and $d^{rty}_j(t)$ disturbances, guaranteeing $\lim_{j \to +\infty} ||u_d(t) - u_j(t)||_\lambda \leq b_u$ with $b_u \geq 0$.

[1]Note that u_d is unique, and that both x_d, and u_d are unknown and required only to theoretically prove the convergence of the method.

III. SOLUTION

This section is dived into four parts. Firstly, we present the employed control law. Secondly, we report well-known results for this iterative control. Third, we propose the main result of this paper, i.e., a robust convergence condition for the control law (3). This converge condition is able to cope with state repetitive and nonrepetitive disturbances, i.e., Def. 1 and 3. Finally, the fourth section extends the main result considering also the presence of time repetitive and nonrepetitive disturbances, i.e., Def. 2 and 4.

A. ITERATIVE CONTROL LAW

In this paper we employ an ILC control law, which is purely feedforward. This has already been widely used in literature, for example in [9] and [30], achieving G1). Recalling the system (1)-(2) and the assumptions A1)-A5), we employed control law is

$$u_{j+1}(t) = u_j(t) + \Gamma_j(t)\, {}^r e_j(t)\ , \tag{3}$$

where $\Gamma_j(t) \in \mathbb{R}^{m \times m}$ is the time and iteration varying learning gain and the error signal ${}^r e_j(t) \in \mathbb{R}^m$ is defined as

$$\begin{aligned} {}^r e_j(t) &\triangleq \sum_{i=0}^{r} \Upsilon_i \left(y^{(i)}_d(t) - y^{(i)}_j(t) \right) \\ &= \underbrace{\sum_{i=0}^{r} \Upsilon_i \left(L^i_{f_n} h(x_d) - L^i_{f_n} h(x_j) \right)}_{\Phi(x_j, x_d)} \\ &\quad + \Upsilon_r \left(D(x_d) u_d - D(x_j) u_j \right)\ , \end{aligned} \tag{4}$$

where $\Upsilon_i \in \mathbb{R}^{m \times m}, \Upsilon_i \succ 0, \forall i = 0, \dots, r$ are tunable control gain matrices, which affect the convergence velocity [9]. The *initial guess* $u_0(t) \in \mathbb{R}^m$ of the iterative control law (3) can be arbitrarily chosen.

Assuming that the measurements of $y^{(i)}_j(t)$ for $i = 0, \cdots, r$ can be easily obtained through sensors, for each iteration j and time instant $t \in [0, t_f]$, the control law (3) requires $(r+1)(m^2 + m)$ operations. In the case that the derivative measurements are not available the method complexity increases depending on the adopted algorithm.

It is instrumental for the derivation of the method to introduce the following definition.

Definition 5. *If for any initial guess $u_0(t) : [0, t_f] \to \mathbb{R}^m$, the iterative control law* (3) *converges to $u_d(t) : [0, t_f] \to \mathbb{R}^m$ in such a way $||u_d(t) - u_j(t)||_\lambda = 0$ when $j \to +\infty$, then* (3) *is said to be convergent.*

Lemma 1. *If the control law is convergent (Def. 5), then the error* (4) *is such that $||{}^r e_j(t)||_\lambda \to 0$ when $j \to +\infty$.*

Proof. Recalling assumptions A3) and A5), i.e., no shift in the initial condition and the feasibility of the desired trajectory, the proof is trivial. □

B. STATE OF THE ART

A sufficient convergence condition [30] for the controller (3), which we call not-disturbed (ND) convergence condition, is

$$\left|\left|I_m-\Gamma_j(t)D(x_j)\right|\right| < 1\ ,\forall j\in U,\forall x\in\mathbb{R}^n,\forall t\in[0,t_\mathrm{f}]\ . \quad (5)$$

If (5) is verified, then the iterative process is guaranteed to convergence. This occurs also in presence of state-repetitive disturbances (Def. 1), see, e.g., [7], [31]. Indeed, considering the system (1), the state-persistent disturbances $d^{\mathrm{px}}(x_j(t))$ can be included in the vector $f_\mathrm{n}(x_j(t))$, which is still Lipschitz. For this reason, in the following, without loss of generality, we can directly consider the disturbed drift vector

$$f(x_j(t)) \triangleq d^{\mathrm{px}}(x_j(t))+f_\mathrm{n}(x_j(t))\ . \quad (6)$$

It is worth noting that, the Lipschitz constant of $f(x_j(t))$ is still $\bar{f}\in\mathbb{R}$, i.e., assumption A4).

Additionally, (5) can also deal with both time-nonrepetitive and time-repetitive disturbances (Def. 2 and 4). However, in this case the iterative process will not have a perfect convergence as in Def. 5, but it will be bounded, i.e., $\left|\left|u_\mathrm{d}(t)-u_j(t)\right|\right|_\lambda \le b_\mathrm{u}$, with $b_\mathrm{u}>0$ finite, see, e.g., [11].

On the other hand, (5) does not guarantee the so-called control contraction when state-nonrepetitive disturbances (Def. 3) occur. Therefore, the main contribution of this work is to propose a robust convergence condition (D), which extends (5). This is presented in the following section.

C. MAIN RESULT: STATE-NONREPETITIVE DISTURBANCES

For the sake of clarity, let us define what follows.

Definition 6. *Let $U\equiv\mathbb{N}$ be the iteration set. U is such that $U=T\cup V$, where V contains all that iteration j such that* (5) *is not fulfilled, while $T=U-V$.*

The following Theorem represents the main result of this paper. It enables the controller (3) to cope with state-nonrepetitive disturbance such as in Def. 3, achieving G2).

Theorem 1. *Let us consider the system in the form* (1)-(2) *with $d^{\mathrm{pt}}(t)\equiv 0_{n\times1}$, $d^{\mathrm{rt}}_j(t)\equiv 0_{n\times1}$, $d^{\mathrm{rty}}_j(t)\equiv 0_{m\times1}$, and let $y_\mathrm{d}(t)\in\mathbb{R}^m$ be the desired output trajectory. Let $N\in\mathbb{N}$ be a finite constant. Under assumptions A1)-A5), if the learning gain $\Gamma_j(t)\in\mathbb{R}^{m\times m}$ satisfies*

$$\prod_{i=j}^{j+N-1}||I_m-\Gamma_i(t)\Upsilon_r D(x_i)|| \le \prod_{i=j}^{j+N-1}\rho_i<1\ , \qquad \forall j=sN\in U,\ s\in\mathbb{N},\forall t\in[0,t_\mathrm{f}]\ . \quad (7)$$

then, the control law (3) *is convergent (Def. 5), i.e., $\left|\left|{}^re_j(t)\right|\right|_\lambda\to 0$, when $j\to+\infty$.*

Proof. For the sake of clarity, we omit the time dependency.

Given the control law (3) and (4), we have

$$\begin{aligned}u_\mathrm{d}-u_{j+1} &= \left(I_m-\Gamma_j\Upsilon_r D(x_j)\right)(u_\mathrm{d}-u_j)-\Gamma_j\Phi(x_j,x_\mathrm{d})\\ &\quad+\Gamma_j\Upsilon_r\left(D(x_j)-D(x_\mathrm{d})\right)u_\mathrm{d}\ .\end{aligned} \quad (8)$$

Defining $\delta u_j \triangleq u_\mathrm{d}-u_j$ and $\delta x_j \triangleq x_\mathrm{d}-x_j$, we can write

$$\begin{aligned}\left|\left|\delta u_{j+1}\right|\right| &\le \left|\left|I_m-\Gamma_j\Upsilon_r D(x_j)\right|\right|\left|\left|\delta u_j\right|\right|+\left|\left|\Gamma_j\right|\right|\left|\left|\Phi(x_j,x_\mathrm{d})\right|\right|\\ &\quad+\left|\left|\Gamma_j\right|\right|\left|\left|\Upsilon_r\right|\right|\left|\left|D(x_j)-D(x_\mathrm{d})\right|\right|\left|\left|u_\mathrm{d}\right|\right|\ .\end{aligned} \quad (9)$$

Given (4) and A4), we compute

$$\begin{aligned}\left|\left|\Phi(x_j,x_\mathrm{d})\right|\right| &= \left|\left|\sum_{i=0}^{r}\Upsilon_i\left(L_f^i h(x_\mathrm{d})-L_f^i h(x_j)\right)\right|\right|\\ &\le \sum_{i=0}^{r}\left|\left|\Upsilon_i\bar{\Phi}_i\right|\right|\left|\left|\delta x_j\right|\right| \le (r+1)\Phi_\star\left|\left|\delta x_j\right|\right|\ ,\end{aligned} \quad (10)$$

with $\Phi_\star\in\mathbb{R}$ is such that $\Phi_\star \ge \max_{i=0,\cdots,r}\left\{||\Upsilon_i||\left|\left|\bar{\Phi}_i\right|\right|\right\}$.

Recalling A4), let χ_j be such that $\left|\left|I_m-\Gamma_j\Upsilon_r D(x_j)\right|\right|\le\chi_j$, defining $\mu \triangleq \sup_t\left\{||\Upsilon_r||\left(\left|\left|\Gamma_j\right|\right|\bar{\eta}\,||u_\mathrm{d}||+(r+1)\Phi_\star\right)\right\}$, one has

$$\left|\left|\delta u_{j+1}\right|\right| \le \chi_j\left|\left|\delta u_j\right|\right|+\mu\left|\left|\delta x_j\right|\right|\ . \quad (11)$$

Using again assumption A4), we can write the following inequality for the system (1)

$$\begin{aligned}\left|\left|\delta x_j\right|\right| &\le \int_0^{t_\mathrm{f}}\left(\bar{f}+\bar{g}\,||u_\mathrm{d}(\tau)||\right)\left|\left|\delta x_j(\tau)\right|\right|\\ &\quad+\left|\left|g(x_j(\tau))\right|\right|\left|\left|\delta u_j(\tau)\right|\right|\mathrm{d}\tau\ .\end{aligned} \quad (12)$$

Applying the Gronwall's Lemma to (12), leads to

$$\left|\left|\delta x_j\right|\right| \le \int_0^{t_\mathrm{f}} c_1\left|\left|\delta u_j(\tau)\right|\right|\mathrm{e}^{c_2(t-\tau)}\mathrm{d}\tau\ , \quad (13)$$

where $c_1 \triangleq \sup_t\left\{\left|\left|g(x_j)\right|\right|\right\}$ and $c_2 \triangleq \sup_t\left\{\bar{f}+\bar{g}\,||u_\mathrm{d}||\right\}$.

Substituting (13) in (11), leads to

$$\left|\left|\delta u_{j+1}\right|\right| \le \chi_j\left|\left|\delta u_j\right|\right|+\mu c_1\int_0^t\left|\left|\delta u_j(\tau)\right|\right|\mathrm{e}^{c_2(t-\tau)}\mathrm{d}\tau\ . \quad (14)$$

Computing the $\lambda-$norm of (14), we obtain

$$\left|\left|\delta u_{j+1}\right|\right|_\lambda \le \chi_j\left|\left|\delta u_j\right|\right|_\lambda+\sup_t \mu c_1\int_0^t\mathrm{e}^{(c_2-\lambda)(t-\tau)}\mathrm{d}\tau\left|\left|\delta u_j\right|\right|_\lambda\ . \quad (15)$$

Grouping for $\left|\left|\delta u_j\right|\right|_\lambda$ and solving the integral, leads to

$$\left|\left|\delta u_{j+1}\right|\right|_\lambda \le \left(\chi_j+\frac{\mu c_1\left(1-\mathrm{e}^{(c_2-\lambda)t_\mathrm{f}}\right)}{\lambda-c_2}\right)\left|\left|\delta u_j\right|\right|_\lambda\ , \quad (16)$$

which can be rewritten as

$$\left|\left|\delta u_{j+1}\right|\right|_\lambda \le \left(\chi_j+\nu_j(\lambda)\right)\left|\left|\delta u_j\right|\right|_\lambda \le \rho_j\left|\left|\delta u_j\right|\right|_\lambda\ . \quad (17)$$

Considering $\chi_j<1$, then $\forall c_2\ge 0$, $\exists\lambda\ge 0$ such that $\chi_j+\nu_j(\lambda)<1$, $\forall j\in U$. It is worth mentioning that this proves the ND-condition (5).

On the other hand, the presence of state-nonrepetitive disturbances $d^{\mathrm{rx}}_j(x)$ (Def. 4) affects the constant c_2 in (13), leading to $c_2' \triangleq c_2+\bar{d}^{\mathrm{rx}}_j$. This may lead to a failure in the convergence condition (5). Indeed, $\forall c_2$, and λ (already selected), $\exists\bar{d}^{\mathrm{rx}}_j : \chi_j+\nu_j(\lambda,\bar{d}^{\mathrm{rx}}_j)>1\ ,\forall j\in V$ in (17).

Without loss of generality, we can group (16) by windows of N trials, which contains iterations belonging to both V and T. This leads to

$$||\delta u_{j+N}||_\lambda \leq \prod_{i=j}^{j+N-1} \rho_i \, ||\delta u_j||_\lambda \triangleq P_j \, ||\delta u_j||_\lambda \ , \tag{18}$$

which is a control contraction for hypothesis, i.e., $P_j < 1$.

We substitute all the iterations of the iterative process, and we compute the limit for $j \to +\infty$

$$\lim_{j\to+\infty} ||\delta u_{j+1}||_\lambda \leq \prod_{j=0}^{+\infty} P_j \, ||\delta u_0||_\lambda = 0 \, . \tag{19}$$

The right-hand side of (19) is an infinite product of factors P_j such that $0 \leq P_j < 1$. This implies that $\prod_{j=0}^{+\infty} P_j = 0$. Recalling Lemma 1, we state that $||^r e_j(t)||_\lambda \to 0,\ j \to +\infty$. Thus, the proof is completed. □

Note that, if we choose $N = 1$, convergence condition (7) (D) shrinks into (5) (ND). Conversely, choosing $1 < N < +\infty$, leads to a convergence condition, which is more robust than (5). Indeed, (7) guarantees the convergence even if (5) is not fulfilled for some iterations.

A necessary and sufficient convergence condition for the controller (3) and nonlinear system (1)-(2) is still an open problem. However, restricting the class of nonlinear systems under study, it is possible to obtain the necessary and sufficient convergence condition for the controller (3), as proven in the following Theorem.

Theorem 2. *Under the same assumption of Theorem 1, let $D(x)$ be the decoupling matrix, such that $D(x) = D$, with D constant matrix such that $\eta = ||D|| \in \mathbb{R}$. Then, (7) is the necessary and sufficient convergence condition for the control law (3).*

Proof. **Sufficiency.** We refer to Theorem 1.

Necessity. By contradiction, let us assume that $||^r e_j(t)||_\lambda \to b \geq 0$, when $j \to +\infty$.

Recalling (4), and A4), leads to

$$||^r e_{j+N}|| \leq (r+1)\Phi_\star \, ||\delta x_{j+N}|| + \eta \, ||\delta u_{j+N}|| \, . \tag{20}$$

Defining $\bar{\Phi}_\star \triangleq (r+1)\Phi_\star$ and substituting (12)-(18) into (20), one has

$$\begin{aligned} ||^r e_{j+N}||_\lambda &\leq (\bar{\Phi}_\star + \nu_{j+N} + \eta) \, ||\delta u_{j+N}||_\lambda \\ &\leq (\bar{\Phi}_\star + \nu_{j+N} + \eta) P_j \, ||\delta u_j||_\lambda \, . \end{aligned} \tag{21}$$

Since $||^r e_j||_\lambda \to b \in \mathbb{R}^+,\ j \to +\infty$, then $P_j \geq 1$ for some j, which is absurd ($P_j < 1 \ \forall j \in U$). Thus $||^r e_j(t)||_\lambda \to 0$, and the proof is completed. □

Since the windows N is not known a priori, (7) results not trivial for a practical interpretation. To have a trivial comparison with a classic convergence condition (ND), i.e., (5), we state what follows.

Corollary 1. *Under the same assumptions of Theorem 1, let $U = V \cup T$ be the iteration set such that $\#T = \infty$ while $\#V < \infty$. A sufficient condition for the convergence of* (3) *is*

$$||I_m - \Gamma_j(t)\Upsilon_r D(x_j)|| \leq \rho_j < 1 \quad \forall j \in T \ , \forall t \in [0, t_f] \, . \tag{22}$$

Proof. We here report only a sketch of it. Recalling (17), we substitute all the previous trials, we split the products, and we compute the limit

$$\lim_{j\to+\infty} ||\delta u_{j+1}||_\lambda \leq \lim_{j\to+\infty} \prod_{j\in V} (\chi_j + \nu_j) \prod_{j\in T} (\chi_j + \nu_j) \, ||\delta u_0||_\lambda \ , \tag{23}$$

in which $\prod_{j\in T} (\chi_j + \nu_j) = 0$ and $\prod_{j\in V} (\chi_j + \nu_j) = \nu_\star \in \mathbb{R}^+ \backslash \{+\infty\}$. The proof is completed. □

We tackle the goal G3) with the following Proposition.

Proposition 1. *Under the same assumptions of Theorem 1, and given a window N of iterations, let N_V and N_T be two sets such that $N = \#N_V + \#N_T$. The two sets N_V and N_T include the iteration indexes j where a state-nonrepetitive disturbance occurs or not, respectively.*

Let be $A \triangleq 1/\prod_{j\in N_T} \rho_j$, $1 \leq A < +\infty$ and let the learning gain $\Gamma_j(t)$ equal to

$$\Gamma_j(t) = \varepsilon \Upsilon_r^{-1} D^{-1}(x_j) \ , \forall t \in [0, t_f], \varepsilon \in (0, 1], \forall j \in U \, . \tag{24}$$

For any iteration window N, the D-condition (7) *holds if the nonrepetitive disturbances are such that*

$$d_\star^{rx} = \max_{j\in N_V} \{\bar{d}_j^{rx}\} < \lambda - c_2 - W\left(\frac{t_f c_1 \mu}{\sqrt[\#N_V]{A}} \exp\left(\frac{\left(\frac{\varepsilon}{c_1\mu} + 1\right) t_f c_1 \mu}{\sqrt[\#N_V]{A}}\right)\right) \frac{1}{t_f} - \frac{\varepsilon}{c_1 \mu} + 1 \, , \tag{25}$$

where W is the Lambert function [32], $\lambda \in \mathbb{R}^+ \backslash \{+\infty\}$, and μ, c_1, c_2 are respectively defined in (11) *and* (13).

Proof. Since we assumed that (7) holds true, recalling (16) and Def. 6, we can write

$$\prod_{j\in N_T} \rho_j \prod_{j\in N_V} \rho_j = A^{-1} \prod_{j\in N_V} \rho_j < 1 \, , \tag{26}$$

where $A^{-1} < 1$ and $\prod_{j\in N_V} \rho_j \in [1, +\infty)$.

Substituting (24) into (26), and computing $d_\star^{rx} = \max_{j\in N_V} \{d_j^{rx}\}$, yield to

$$\prod_{j\in N_V} \rho_j = \left(\varepsilon + \frac{\mu c_1 \left(1 - e^{(c_2 - \lambda - d_\star^{rx}) t_f}\right)}{\lambda - c_2 - d_\star^{rx}}\right)^{\#N_V} < A \, . \tag{27}$$

Note that, we are looking for $d_\star^{rx} \in \mathbb{R} \backslash \{\infty\}$, which satisfies (27). Then, after mathematical manipulation, and defining $\zeta \triangleq \lambda - c_2 - d_\star^{rx}$, one has $e^{-\zeta t_f} c_1 \mu > -\sqrt[\#N_V]{A}\zeta - \varepsilon + c_1 \mu$, whose solution is (25). □

In practice, (25) is difficult to apply, but it guarantees an upper bound w.r.t. the iteration frequency for any state-nonrepetitive disturbances.

Remark 2. *The control law* (3) *depends on the control gains $\Upsilon_i \in \mathbb{R}^{m\times m}$, for $i = 0,\cdots,r$. These directly multiply the derivative of the output. Large values could speed up the convergence of the method. However, the magnitude of the gains should be proportional to the reliability of the measurements. i.e., inaccurate measurements should be multiplied by low gains. Moreover, in practical applications, the control action could exceed the actuators physical limits and, eventually, damage the system.*

D. OTHER RESULTS: ALL DISTURBANCES

In this section, we analyze the presence of also the time-nonrepetitive and repetitive disturbances (Def. 2 and 4), achieving G4). As discussed in Sec. III-B, these disturbances do not affect (7), although, they lead to a bounded error [28] and [18]. The following Theorem extends Theorem 1 w.r.t. all disturbances under analysis, relaxing also A3).

Theorem 3. *Let us consider the system in the form* (1)-(2), *and let $y_{\rm d}(t) \in \mathbb{R}^m$ be desired output trajectory.*

Under assumptions A1), A2), A4), A5), let us consider the initial condition such as $x_j(0) = x_{\rm d}(0) + l_j, \forall j \in U$, with $\sup_j ||l_j|| \le b_{\rm l} < +\infty$. Let us assume that the time-nonrepetitive disturbances $d_j^{\rm rty}(t) \in \mathbb{R}^m$ (Def. 4) is time differentiable for r times with bounded derivatives, namely $\bar{d}_0^{\rm rty},\cdots,\bar{d}_r^{\rm rty} \in \mathbb{R}\backslash\{+\infty\}$.

If $\Gamma_j(t) \in \mathbb{R}^{m\times m}$ satisfies (7), then the controller (3) is such that $||u_{\rm d}(t) - u_j(t)||_\lambda \le b_{\rm u}$, when $j \to +\infty$ with $b_{\rm u} \in \mathbb{R}_+\backslash\{+\infty\}$.

Proof. The presence of time-repetitive and nonrepetitive disturbances modify (11) such as

$$||\delta u_{j+1}|| \le \chi_j ||\delta u_j|| + \mu ||\delta x_j|| + ||\Gamma_j|| \bar{d}^{\rm rty} , \quad (28)$$

with $\bar{d}^{\rm rty} = (r+1)\max\{||\Upsilon_0||\bar{d}_0^{\rm rty},\cdots,||\Upsilon_r||\bar{d}_r^{\rm rty}\}$.

Now, let us recall (13), which becomes

$$||\delta x_j|| \le b_{\rm l}{\rm e}^{c_2 t} + \int_0^{t_{\rm f}} (c_1 ||\delta u_j(\tau)|| + \bar{d}_j^{\rm rx}){\rm e}^{c_2(t-\tau)}{\rm d}\tau , \quad (29)$$

where $\bar{d}_j^{\rm rx} = \max_t\{d_j^{\rm rx}(t)\}$. Then, with analogous calculation from (14)-(18), we derive

$$||\delta u_{j+N}||_\lambda \le \prod_{i=j}^{j+N-1} \rho_i ||\delta u_j||_\lambda + \sum_{k=1}^{N-1} \prod_{i=j}^{j+N-k} \rho_i \bar{d} + \bar{d}_{j+N} , \quad (30)$$

with $\bar{d} \triangleq \sup_j\{\bar{d}_j\} = \sup_j\{||\Gamma_j||\bar{d}^{\rm rty} + \mu b_{\rm l} + \mu \bar{d}_j^{\rm rx}\nu(\lambda)\} < +\infty$, and $\bar{d}_{j+N} \triangleq \sup_j\{\bar{d}_{j+N}\} = \sup_j\{||\Gamma_{j+N}||\bar{d}^{\rm rty} + \mu b_{\rm l} + \mu \bar{d}_{j+N}^{\rm rx}\nu(\lambda)\} < +\infty$.

Computing the limit for $j \to +\infty$, using (7), and rearranging (30) by splitting into N iteration products, lead to

$$\lim_{j\to+\infty} ||\delta u_{j+N}||_\lambda \le \prod_{j=1}^{+\infty} P_j ||\delta u_0||_\lambda + \sum_{j=1}^{+\infty} P_j \bar{d} + \bar{d}_N , \quad (31)$$

where $\bar{d}_N$ is bounded because it is a finite sum of N bounded variables.

Recalling (7), and defining $\overline{P} \triangleq \sup_j \max_t P_j$, one has

$$\lim_{j\to+\infty} ||\delta u_{j+N}||_\lambda \le \frac{1}{1-\overline{P}}\bar{d} + \bar{d}_N \triangleq b_{\rm u} < +\infty . \quad (32)$$

The proof is completed. □

IV. VALIDATION

We validate the effectiveness of the D-condition through simulations, using MATLAB. Firstly, we simulate a 2 DoFs underactuated compliant robot, namely $R\overline{R}$, composed of two elastic joints, where only the first one is actuated. Secondly, we test the method on a *Franka Emika Panda* robot equipped with elastic joints.

The dynamic model is used for simulating the system and for tuning the gain $\Gamma_j(t)$ of the controller (3). The gains Υ are chosen depending on the system, while $\varepsilon = 0.9$. The initial guess u_0 is chosen equal to the constant torque able to maintain the robot in the starting position of the trajectory $y_{\rm d}(0)$, i.e., solving $f_{\rm n}(x_{\rm d}(0)) + g(x_{\rm d}(0))u_0(t) = 0$.

To quantify the tracking performance, we use as a metric the root mean square (RMS) of the norm of each component of the output error, showing that the D-condition (7) extends the ND-condition (5). The learning is executed until the RMS error reaches a value of 0.001rad.

A. TWO DOFS UNDERACTUATED ROBOT: $R\overline{R}$

We simulate the dynamics of a two DoFs underactuated arm with elastic joints. We refer to [9] for a more exhaustive treatment of the system dynamics. Let $m = 0.55$ kg, $J = 0.001$ kgm^2, $l = 0.085$ m, $a = 0.089$ m, and $d_v = 0.3$ Nms/rad be the mass, inertia, length, center of mass distance, and damping of each link, respectively. The stiffness of each link is tested in two configurations: Soft, i.e., $k = 1$ Nm/rad, and Stiff, i.e., $k = 3$ Nm/rad. For the sake of clarity, let us recall that the state $x \in \mathbb{R}^4$ of the robot is $x = [x_1, x_2, x_3, x_4]^\top$, where x_1 and x_2 are the joint positions, while x_3 and x_4 are the joint angular velocities.

To test the robustness of the method, we design the learning gain Γ_j using a model whose parameters are different from the nominal one. In particular, the second link parameters $\tilde{m}_2, \tilde{J}_2, \tilde{l}_2$, and $\tilde{a}_2$ are decreased by 50%. This is a state-repetitive disturbance $d^{\rm px}$ in (1). Additionally, we test the control algorithm simulating measurement noise $d_j^{\rm rty}(t)$, external disturbances, and delays in the controller $u_j(t)$, which can be both modeled as state-nonrepetitive disturbances $d_j^{\rm rx}(x_j(t))$ in (1).

The chosen output function $h(x) \in \mathbb{R}$ is the absolute angle of the robot tip i.e., $y = x_1 + x_2$, which leads to a relative degree $r = 2$ iff ([9])

$$D(x) = L_g L_{f_{\rm n}} h(x) = \frac{-b_2 \cos(x_2)}{\det{\rm M}(x)} \ne 0 , \quad (33)$$

where $b_1 = m_2 a_1^2 + m_1 l_1^2 + J_1$, $b_2 = m_2 l_2^2 + J_2$, $b_3 = a_1 l_2 m_2$ and $\det{\rm M}(x) = b_1 b_2 + b_3^2 \cos x_2 \ne 0, \forall x \in \mathbb{R}^n$.

The desired trajectory is a minimum jerk signal that starts from the initial position $y_0 = 0$ and reaches the final one $y_\mathrm{f} = \frac{\pi}{4}$ in $t_\mathrm{f} = 10$s, i.e.,

$$y_\mathrm{d}(t) = y_\mathrm{f}\left(10\left(\frac{t}{t_\mathrm{f}}\right)^3 - 15\left(\frac{t}{t_\mathrm{f}}\right)^4 + 6\left(\frac{t}{t_\mathrm{f}}\right)^5\right) . \quad (34)$$

To fulfil (7), we choose a constant learning gain

$$\Gamma = -\varepsilon \tilde{\det \mathrm{M}}/\tilde{b}_3 \,, \quad (35)$$

with $\tilde{b}_3 = a_1\tilde{l}_2\tilde{m}_2$ and $\tilde{\det\mathrm{M}} = b_1\tilde{b}_2 - \tilde{b}_3^2$.

In each trial, the starting configuration is $x_j(0) = 0_{4\times1}\ \forall j \in U$, and the control gains are $[\Upsilon_0, \Upsilon_1, \Upsilon_2] = [80, 5, 1]$.

1) Soft Configuration

In presence of particularly low stiffness, during the robot motion, the second link position x_2 reaches $x_2 = \pi/2$. Thus, (33) vanishes leading to a variation of the relative degree. This variation causes a failure of the convergence condition (5), and no conclusions on the convergence of the iterative method can be drawn. However, this simulation shows that using the gain (35), we can guarantee the convergence thanks to (7).

We test the same task in two different conditions:

- *D - Model*: we design the learning gain using (35). The disturbances are due to model uncertainties and a change in the relative degree, namely $d^\mathrm{pt}(t)$.
- *D - Noise*: we employ (35), and, in addition to the issues of the *D - Model* case, we inject Gaussian noise into the system simulating the presence of time-nonrepetitive disturbances $d_j^\mathrm{rty}(t)$. The mean value of the Gaussian noise is equal to 0, the standard deviation equal to 10^{-3} on the position measurements, and 10^{-5} on the velocity measurements.

Thus in the *D - Noise* scenario, recalling (1)-(2), and (6), the simulated system can be written as

$$\begin{cases} \dot{x}_j(t) = f(x_j(t)) + g(x_j(t))u_j(t) + d^\mathrm{pt}(t) + d_j^\mathrm{rt}(t) & (36) \\ y_j(t) = h(x_j(t)) + d_j^\mathrm{rty}(t) \ . & (37) \end{cases}$$

Finally, in the *D - Model* scenario, we have $d_j^\mathrm{rty}(t) \equiv 0_{m\times1}$, and $d^\mathrm{pt}(t) \equiv d_j^\mathrm{rt}(t) \equiv 0_{2n\times1}$ in (36)-(37).

Fig.1 reports the simulation results, where at trials $j = 6, 8$, x_2 is $x_2 = \pi/2$. Fig.1(a) shows the tracking performance at the last iteration, while Fig.1(b)-1(c) depict the error evolution over iterations.

2) Stiff Configuration

We test the same task in four different conditions:

- *ND*: we use the nominal model in (24), where $D(x_j)$ is computed as (33).
- *D - Model*: we design the learning gain using (35). The disturbances are due to model uncertainties, i.e., $d^\mathrm{px}(t)$.
- *D - Force*: we employ (35), and, in addition to the issue of the *D - Model* case, we simulate the presence of an external force due to an interaction between the robot and the environment, which occurs at trails $j = 8, 12$. This is a state-nonrepetitive disturbance $d_j^\mathrm{rx}(x_j(t))$, which is mapped at the joint level with $\bar{d}_j^\mathrm{rx} = 0.5$ Nm at $t = 5$ s.
- *D - Delay*: we employ (35), and, in addition to the issue of the *D - Model* case, we simulate the presence of a 1 s delay in the control action. This occurs at trails $j = 8, 12$ and it can be modeled as state-nonrepetitive disturbance $d_j^\mathrm{rx}(x_j(t))$.

Thus, in the *D - Force and Delay* scenarios, recalling (1) and (6), the simulated system can be written as

$$\dot{x}_j(t) = f(x_j(t)) + g(x_j(t))u_j(t) + d_j^\mathrm{rx}(x_j(t)) + d^\mathrm{pt}(t) \ . \quad (38)$$

Finally, in the *D - Model* scenario, we have $d^\mathrm{pt}(t) \equiv d_j^\mathrm{rx}(x_j(t)) \equiv 0_{2n\times1}$ in (38).

Fig. 2 reports the simulation results. Fig. 2(a) depicts the tracking performance at the last iteration, while Fig. 2(b)-Fig. 2(c) show the error evolution over iterations.

It is worth mentioning that, taking $N = 5$, $\lambda = 1.8$, $A = 1.52$, $\mu = 3\mathrm{e}-4$, $c_2 = \pi/4$ and $c_1 = 26$, (25) holds. In particular we have that in the *D - Force* scenario $d_\star^\mathrm{rx} = 0.5$, while in the *D - Delay* scenario $d_\star^\mathrm{rx} = 0.4$.

B. SERIAL MANIPULATOR

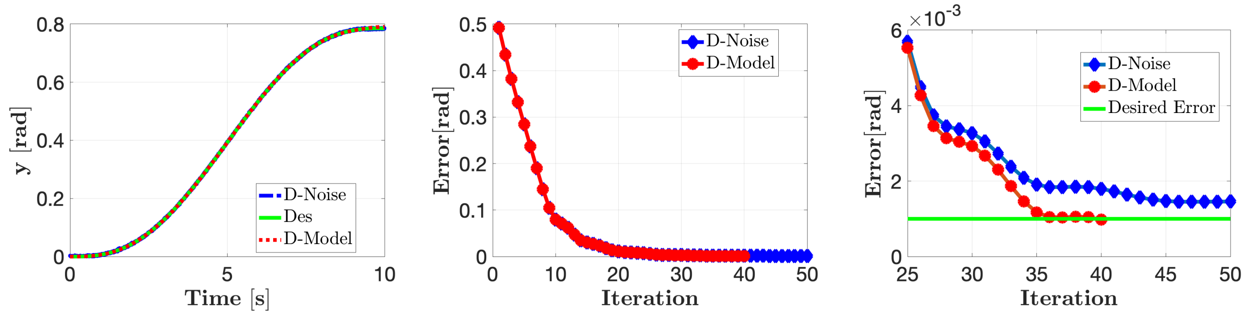
We simulate a 7-DoFs *Franka Emika Panda*[2] manipulator adding a joint stiffness matrix $K = \mathrm{diag}\{5,5,5,3,3,3,3\}\,1\mathrm{e}2$ and a joint damping matrix $F = \mathrm{diag}\{10,10,10,5,5,5,5\}$. Additional details on the dynamics model of the robot can be found in [33].

We design a Cartesian trajectory $(X - X_0)^2 + (Y - Y_0)^2 + Z_0^2 = R^2$, where $[X_0, Y_0, Z_0]^\top \in \mathbb{R}^3$ is the Cartesian starting position of the robot and $R = 0.1$ m is the radius of the circumference. Solving the inverse kinematic leads to the desired time evolution of the joints, i.e., $y_\mathrm{d}(t) = [\mathtt{ones}(1,7), \mathtt{zeros}(1,7)]x_\mathrm{d}$, in such a way that the relative degree r is $r = 2$, [34]. We indicate the nominal inertia matrix of the robot as $M(q)$ and its model with $\tilde{M}(q) = 0.9M(q)$. This is a state-repetitive disturbance d^px in (1). Note that both $M(q), \tilde{M}(q) \succ 0$. The control gains are $\Upsilon_0 = \hat{M}^{-1}(q)\mathrm{diag}\{5,5,3,3,7,7,10\}\,1\mathrm{e}1$, $\Upsilon_1 = \hat{M}^{-1}(q)\mathrm{diag}\{3,3,3,5,5,5,5\}$, $\Upsilon_2 = 0.1\mathrm{diag}\{\mathrm{ones}(1,7)\}$, where $\hat{M}$ is either the nominal or the perturbed inertia matrix depending on the case under study.

We test the same task in three different conditions:

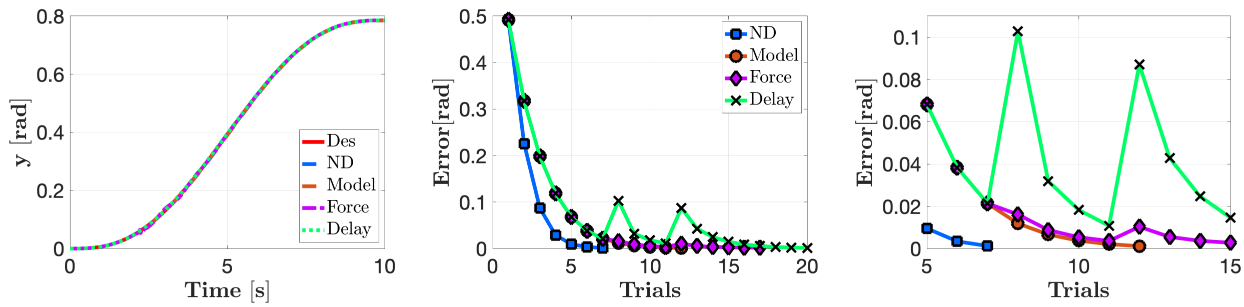
- *ND*: we use (24), where $D(x_j) = M^{-1}(q_j)$.
- *D - Data Loss*: we design the learning gain such as $\Gamma_j(t) = \varepsilon\tilde{M}(q_j)$, which is a model uncertainty, namely $d^\mathrm{pt}(t)$. Additionally, at trials $j = 4, 6$, we simulate a complete loss of joint position data, i.e., $\Gamma_{j+1} = \varepsilon\tilde{M}(q_0)$ leading to a failure of (5). The loss of data can be modeled as a state-nonrepetitive disturbance $d_j^\mathrm{rx}(x_j(t))$.
- *D - Delay*: in addition to designing the learning gain such as in the *D - Data Loss* case, we simulate the presence of a delay of 0.8 s in the control action of

[2] https://www.franka.de/



(a) Output tracking performance. (b) Error evolution over iteration. (c) Zoom: error evolution over iteration.

FIGURE 1. $R\bar{R}$ simulation results in the Soft Configuration.



(a) Output tracking performance. (b) Error evolution over iteration. (c) Zoom: error evolution over iteration.

FIGURE 2. $R\bar{R}$ simulation results in the Stiff Configuration.

the joints 1,3,6,7 at the trails $j = 8, 12$. This leads to a failure of (5). The delay can be seen as state-nonrepetitive disturbance $d_j^{\mathrm{rx}}(x_j(t))$.

Note that the learning gain $\Gamma_j(t) \in \mathbb{R}^{m\times m}$ is nonlinear.

Thus, in the *D - Delay and Data Loss* scenarios, recalling (1) and (6), the simulated system can be written as

$$\dot{x}_j(t) = f(x_j(t)) + g(x_j(t))u_j(t) + d_j^{\mathrm{rx}}(x_j(t)) + d^{\mathrm{pt}}(t) \ . \tag{39}$$

Finally, in the *ND* scenario, we have $d^{\mathrm{pt}}(t) \equiv d_j^{\mathrm{rx}}(x_j(t)) \equiv 0_{2n\times 1}$ in (38).

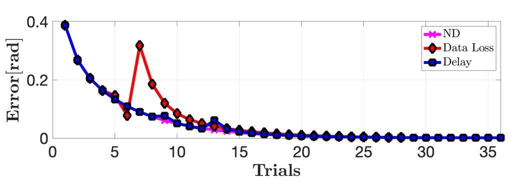
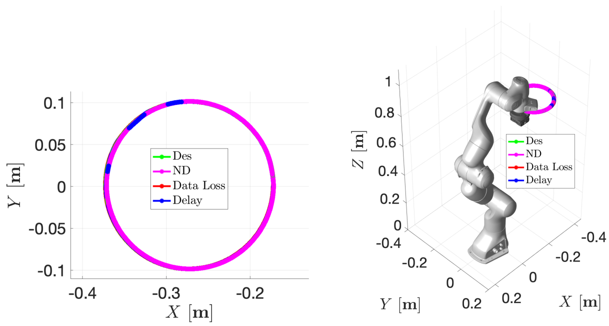


FIGURE 3. *Franka Emika Panda*: error evolution over iteration.

Fig 3 shows error evolution over iterations, while Fig 4(a) compares the Cartesian trajectory executed with the ND and the D conditions at the last trail. Finally, Fig 4(b) shows a 3-D view of the robot at the end of the learning phase.

It is worth mentioning that, taking $N = 10$, $\lambda = 1.6\ 1e3$, $A = 2.1$, $\mu = 0.078$, $c_2 = 1.9$ and $c_1 = 195$, (25) holds. In particular we have that $d_\star^{\mathrm{rx}} = 770$ in both D scenarios (7).



(a) Output tracking performance. (b) Cartesian execution.

FIGURE 4. *Franka Emika Panda* simulation.

V. DISCUSSION

Results show that the proposed method improves the tracking error between the first and the last iteration G1), reaching the desired tracking error value (0.001rad) in presence of state-repetitive and state-nonrepetitive disturbances (Theorem 1, goal G2)) both in case of underactuated (Fig. 1(b)-2(b)) and MIMO systems (Fig. 3). Quantification of the robustness of the method is also presented (Proposition 1, goal G3)). As expected, the error convergence is not achieved in the case of time-repetitive and nonrepetitive disturbances (Fig. 1(c)), where a bounded error is obtained (Theorem 3, goal G4)).

If the employed model is exact, and there are no disturbances, the converge is smooth, fast, and exponential Fig. 2(b)-Fig. 3. On the other hand, as expected, the presence of state-nonrepetitive disturbances leads to an increment of the error for some iterations, Fig. 1(b)-Fig. 2(b)-Fig. 3, leading to a non-monotonic convergence. However, thanks to the fulfillment of the D-condition we proposed, the controller is able to achieve the same tracking performance (goal G2)) Fig. 2(a)-Fig. 4(a). This proves that the D-condition is more robust w.r.t. the original ND one. Indeed, the D-condition obtains the minimization of the error while dealing with the incorrect contribution added to the control input, achieving the same tracking performance as the ND-condition.

VI. CONCLUSION AND FUTURE WORK

In this paper, we tackled the problem of trajectory tracking for continuous-time nonlinear systems affected by disturbances. We define different classes of disturbances. The goal was to obtain a controller able to achieve good tracking performance even in presence of state-nonrepetitive disturbances. We proposed and proved a convergence condition for a class of iterative learning controllers. The algorithm is purely feedforward, and it copes with nonlinear systems with a generic and fixed relative degree. The proposed method is robust both to repetitive and nonrepetitive disturbances. Additionally, we presented an upper bound of the disturbance amplitude that can be dealt with. Finally, we validated the proposed method through simulations using an underactuated compliant arm and *Franka Emika Panda* robot, both subjected to different types of disturbances.

Future work will investigate the robustness of the iterative framework from both a theoretical and an experimental point of view. We will combine feedforward and feedback terms and design switching policies depending on the system relative degree. Additionally, the employed control law (3) is based only on the output measurements. Future work will rely on state-observers [35] to design a control law employing the knowledge of the whole state. Finally, from a more experimental point of view, we will implement the algorithm on a real soft continuum prototype and medical image encryption [36] both with disturbances.

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