

# Contact and Grasp Robustness Measures: Analysis and Experiments

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## Abstract

In this paper we discuss some aspects related to the practical assessment of the quality of a grasp by a robotic device on objects of unknown shape, based on sensorial feedback from tactile and force sensors on the hand. We briefly discuss the concept of contact robustness and of grasp robustness, pointing out that the former is an easily computable but overconservative sufficient condition for the latter. Some experimental results on a simple gripper, the so-called “Instrumented Talon”, are reported as an illustration.

## 1. Introduction

This paper presents procedures for the assessment of the quality of grasps by robotic hands. The interest of having a good measure of the quality of the grasp is twofold: during planning of a manipulation sequence, it allows to optimize the positioning of the hand with respect to the object to be grasped, and the grasping forces; during the execution of a grasping task, such measure can be used as a performance index according to which local optimization techniques can be used in order to react, at least sub-optimally, to external disturbances and modelling errors.

In the literature, there is a wide interest in the problem of planning good grasps. In [2] the quality criteria of grasp are based on the minimization of the sum of the maximum finger force ( $L_\infty$  metric) and of the total finger force ( $L_1$  metric). In [3] the goodness of a grasp is defined in the space of object wrenches and is given as the radius of the largest closed ball, centered in the origin of the space, contained in the set of all the possible wrenches that can be resisted

by applying at most unit forces at contacts. An optimality criterion depending on the specific task to be executed has been addressed by Li and Sastry in [4]. In [9] the approach to the analysis of the grasp quality consists of looking at the distance from the vector of contact forces to the nearest contact constraint, suggesting that the farther is the worst-case finger force from violation of a constraint, the better the grasp is. This approach is very intuitive and has been widely used in literature. Naturally, the choice of internal forces by the controller affects the grasp quality, and one is led to consider, for each grasping configuration, a quality measure related to the best force distribution that an optimizing grasp force controller can possibly achieve.

In this paper, we build upon previous contributions by analysing more closely two aspects that influence the concept of “good” grasp. A first peculiarity of our analysis is related with the fact that enveloping (alias “power”, or “whole-hand”) grasping is explicitly considered. In such style of grasping, not only the fingertips, but also the inner parts of the gripper are exploited in order to achieve more robust hold on the object. This fact implies that contact constraints on the object may be imposed by members of the robotic hand which only enjoy limited mobility, and are therefore not able to exert arbitrary forces at the contact at will.

Secondly, it is observed that in most practical grasp, the set of contact constraints is redundant, in the sense that violation of some of them (slipping or detaching the contact) may well not imply mobilization of the object in the grasp. We therefore suggest that “contact robustness” measures are distinguished from

“grasp robustness” measures, where the former are related to distances from the violation of any contact constraint, while the latter is concerned with actually overcoming the immobilization constraint of the object. It thus turns out that contact robustness is an easily computable but overconservative sufficient condition for grasp robustness, which is the property of actual concern in grasping.

Experimental activity on a testbed comprised of a simple enveloping gripper known as the “Instrumented Talon” developed at the MIT AI Lab is finally described.

## 2. Contact Model

When the manipulation system is modeled by rigid-bodies, the  $i$ -th contact imposes that some components of the relative velocity between the surfaces are zero. Mathematically, this can be written as  $\mathbf{H}_i ({}^h \dot{\mathbf{c}}_i - {}^o \dot{\mathbf{c}}_i) = \mathbf{0}$ , where  $\mathbf{H}_i$  is a constant selection matrix depending on the physical model assumed for the  $i$ -th contact and  ${}^h \mathbf{c}_i, {}^o \mathbf{c}_i$  are vectors locally describing the posture of reference frames attached to the surface of the hand and of the object, respectively. For the sake of simplicity, in this paper we focus on hard-finger contact models only. Small displacements of the contact frames can be expressed as a linear function of small displacements of the object  $\delta \mathbf{u}$  and of the joints  $\delta \mathbf{q}$ , respectively, as  $\delta {}^o \mathbf{c}_i = \tilde{\mathbf{G}}_i^T \delta \mathbf{u}$  and  $\delta {}^h \mathbf{c}_i = \tilde{\mathbf{J}}_i \delta \mathbf{q}$ . In juxtaposed vectorial notation, one has that rigid-body constraints can be summarized by the equation  $\mathbf{H}(\tilde{\mathbf{J}}\delta \mathbf{q} - \tilde{\mathbf{G}}^T \delta \mathbf{u}) = \mathbf{0}$ . Matrix  $\mathbf{G} \stackrel{def}{=} \tilde{\mathbf{G}}\mathbf{H}^T$  is usually referred to as the “grasp matrix”, or “grip transform”, while matrix  $\mathbf{J} \stackrel{def}{=} \mathbf{H}\tilde{\mathbf{J}}$  is called “hand Jacobian”.

As mentioned in the introduction, in this paper we allow for general grasping conditions, including enveloping grasps that exploit kinematically defective links to contact and constrain the object. Kinematic defectivity reflects in the fact that the hand jacobian is not full row rank. It has been shown in previous work of the authors ([7]) that in enveloping grasping, the rigid body model is not adequate to describe unambiguously the system, and in particular its force distribution problem. Accordingly, a more accurate model describing how elastic energy can be stored in the system is necessary. We consider a simplified

model of elasticity in the system, i.e., introduce a set of “virtual springs” at the contact points with characteristic stiffness  $\mathbf{K}_{i_s}$ . This allows us to define a contact force at the  $i$ -th contact as

$$\mathbf{t}_i = \mathbf{K}_{i_s} \mathbf{H}_i (\delta {}^h \mathbf{c}_i - \delta {}^o \mathbf{c}_i), \quad (1)$$

where it is assumed that in the equilibrium configuration it holds  $\mathbf{t}_i = \mathbf{0}$ . Juxtaposing the  $n$  contact force vectors  $\mathbf{t}_i$  is a single vector  $\mathbf{t}$ , one has therefore

$$\delta \mathbf{t} = \mathbf{K}_s (\mathbf{J} \delta \mathbf{q} - \mathbf{G}^T \delta \mathbf{u}), \quad (2)$$

where  $\mathbf{K}_s = \text{diag}(\mathbf{K}_{1_s}, \dots, \mathbf{K}_{n_s})$ .

Contact forces are subject to unilateral constraints enforcing the unisense nature of contact forces and Coulomb’s friction law. Letting  $p_{i_k}$  ( $k = x, y, z$ ), be the component of the contact force  $\mathbf{t}_i$  along the  $k$ -axis of the  $i$ -th contact frame fixed to the object, these are written as

$$\text{a) } p_{i_z} \geq 0, \quad \text{b) } \sqrt{p_{i_x}^2 + p_{i_y}^2} \leq \mu_i p_{i_z}. \quad (3)$$

## 3. Contact and Grasp Robustness

Suppose that a robotic hand grasps an object by means of  $n$  contacts and its configuration is of static equilibrium with balance equations:  $\tau = \mathbf{J}^T \mathbf{t}$  and  $\mathbf{w} = -\mathbf{G}\mathbf{t}$ , being  $\tau$  the vector of joint torques and  $\mathbf{w} = [\mathbf{f}^T, \mathbf{m}^T]^T$  the external wrench acting on the object. We introduce the following hypotheses (cf. [7]):

**H1:** the subspace of *under-actuated* object displacements  $\ker(\mathbf{G}^T)$  is void;

**H2:** the manipulation system is asymptotically stabilized in the equilibrium point by a joint-position feedback controller with steady state gain  $\mathbf{K}_p$ .

**H3:** contact points do not change by rolling (this assumption is reasonable whenever the relative curvature is large).

Consider the vector

$$\mathbf{d}(\mathbf{t}) = [(d_{1_c}, d_{1_f}), \dots, (d_{n_c}, d_{n_f})]^T, \quad (4)$$

where  $d_{i_c}$  is the distance of  $\mathbf{p}_i$  from the tangent plane to the object surface at the  $i$ -th contact and  $d_{i_f}$  is the distance of  $\mathbf{p}_i$  from the friction cone. Vector  $\mathbf{d}(\mathbf{t})$  indicates how far the grasp is from violating contact

constraints (3) and plays a fundamental role in the evaluation of the grasp quality measure. For instance, Kerr and Roth [9] base their quality measure of the grasp on the minimum component of the vector  $\mathbf{d}(\mathbf{t})$ .

### 3.1. Contact Robustness

In  $\mathbb{R}^{3n}$ , the inequality  $\|\delta\mathbf{t}\| \leq \|\mathbf{d}(\mathbf{t})\|_\infty$  describes a sphere centered in the equilibrium contact force and provides a sufficient condition on the maximum euclidean norm of contact force perturbations  $\delta\mathbf{t}$  in order to avoid slippage at all contacts. This property is what we call “contact robustness”.

In order to assess contact robustness of a grasp, the limitation of  $\|\mathbf{d}(\mathbf{t})\|_\infty$  expressed in the contact force space, needs to be reflected in the space of external disturbances acting on the object. We will denote such disturbances as  $\delta\mathbf{w}$  (referring again to departures from an equilibrium condition). In the quasi-static setting chosen for this paper, the map from contact forces  $\delta\mathbf{t}$  to object disturbance wrenches  $\delta\mathbf{w}$  can be obtained (via the principle of virtual work) as  $\delta\mathbf{w} = -\mathbf{G}\delta\mathbf{t}$ . What is needed to assess contact robustness is the inverse of such map. It must be noted that, in a rigid-body setting, such inverse is not unique. However, the problem is well posed if finite stiffness in the system is assumed. The force distribution map is in fact the stiffness-weighted pseudo-inverse of the grasp matrix, as discussed e.g. in [6]. If joint positions are stabilized about the equilibrium configuration by a controller whose static gain matrix is  $\mathbf{R}_q$ , we have

$$\delta\mathbf{t} = -\mathbf{G}_K^R \delta\mathbf{w}; \text{ with } \mathbf{G}_K^R \stackrel{\text{def}}{=} \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1} \quad (5)$$

where  $\mathbf{K} \stackrel{\text{def}}{=} \mathbf{K}_s(\mathbf{I} - \mathbf{J}(\mathbf{J}^T\mathbf{K}_s\mathbf{J} + \mathbf{R}_q)^{-1}\mathbf{J}^T\mathbf{K}_s)$  is the composite grasp stiffness matrix (see [1]).

In order to make our following arguments independent from measurement units, we assume that the wrench vector  $\delta\mathbf{w}$  is scaled with respect to the nominal value of expected external disturbance wrenches in the task under consideration, such that it results adimensional.

**Proposition 1** *Under the hypotheses H1–H3 and in quasi-static conditions, a given grasp is able to resist any disturbance wrench  $\delta\mathbf{w}$  without violating*

*constraints (3) at any contact point, provided that*

$$\|\delta\mathbf{w}\| \leq \frac{\|\mathbf{d}(\mathbf{t})\|_\infty}{\sigma_{\max}(\mathbf{G}_K^R)} \quad (6)$$

where  $\sigma_{\max}(\mathbf{G}_K^R)$  is the maximum singular value of the  $\mathbf{K}$ -weighted right-inverse  $\mathbf{G}_K^R$ , (5).

According to the above proposition, the right-hand side term of (6) can be defined as the “measure of quasi-static contact robustness”.

*Proof:* From the inverse map (5), the relationship  $\delta\mathbf{t}^T\delta\mathbf{t} = \delta\mathbf{w}^T\mathbf{G}_K^{R^T}\mathbf{G}_K^R\delta\mathbf{w} \leq \|\mathbf{d}(\mathbf{t})\|_\infty^2$  describes the ellipsoid in the wrench space centered in zero and with principal axes  $2\|\mathbf{d}(\mathbf{t})\|_\infty/\sigma_k(\mathbf{G}_K^R)$  of length. The inscribed sphere (6) represents a limit on the euclidean norm of  $\delta\mathbf{w}$  ensuring that all contact constraints hold, notwithstanding the wrench disturbance. The condition on  $\ker(\mathbf{K}\mathbf{G}^T)$  guarantees that the sphere is 6-dimensional.  $\square$

A similar measure of contact robustness has been studied, including dynamics effects, for non-defective manipulators in [10], so that our contribution here is only that of pointing out the role of the stiffness matrix  $\mathbf{K}$  in evaluating contact robustness for enveloping grasps. Furthermore, as observed in [4], the proposed measure is only a partial information on the grasp, as two grasp ellipsoids may share the maximal inscribed sphere, though having different shapes.

An important aspect of grasping is that the force distribution may be improved by suitably modifying internal forces, i.e., contact forces in the nullspace of the grasp matrix  $\mathbf{G}$ . A measure of contact robustness can be useful in applications, that takes into account such possibility of redistributing contact forces. However, as already noted, in enveloping grasps the kinematic defectivity of the gripping mechanism may prevent the actual controllability of internal forces, so that the best policy for redistributing contact forces in the grasp must be confined to modifying only some of the internal forces. According to [6] and [7], the subspace of internal forces that are asymptotically reproducible (hence, quasi-statically controllable) is given by

$$\mathcal{F}_a = \ker(\mathbf{G}) \cap (\text{im}(\mathbf{K}\mathbf{J}) + \text{im}(\mathbf{K}\mathbf{G}^T)) \quad (7)$$

$$= \text{im}(\mathbf{I} - \mathbf{G}_K^R\mathbf{G})\mathbf{K}\mathbf{J}. \quad (8)$$

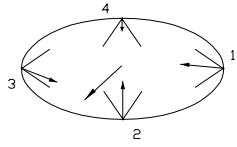


Figure 1. 4-contact grasp

Letting  $\mathbf{E}$  denote a matrix whose columns span  $\mathcal{F}_a$  (and assuming that there is no preload or “jamming” force in the grasp), the general solution to the balance equation  $\mathbf{w} = -\mathbf{G}\mathbf{t}$  is given by  $\mathbf{t} = -\mathbf{G}_K^R \mathbf{w} + \mathbf{E}\mathbf{y}$  where  $\mathbf{y}$  parameterizes controllable internal forces in the basis  $\mathbf{E}$ .

Having included the possibility of “squeezing” harder the object in order to improve the grasp, upper bounds on the intensities of contact forces have to be implemented. Mathematically, this bound can be written in terms of the maximum intensity of the  $i$ -th contact force:

$$\|\mathbf{p}_i\| \leq f_{i,max} \quad \text{with} \quad f_{i,max} > 0, \quad (9)$$

The distance vector  $\mathbf{d}(t)$  is modified accordingly, including as new components the distances  $d_{i,max} = f_{i,max} - \|\mathbf{p}_i\|$ . We hence define a measure of potential contact robustness as

$$\max_{\mathbf{y}} \frac{\|\mathbf{d}(\mathbf{G}_K^R \mathbf{w} + \mathbf{E}\mathbf{y})\|_{\infty}}{\sigma_{max}(\mathbf{G}_K^R)} \quad (10)$$

An efficient algorithm to evaluate the measure above can be found in ([8]).

### 3.2. Grasp Robustness

To illustrate the need to introduce the concept of grasp robustness, consider the planar example of fig. 3.2. For the external wrench and force distribution depicted, the grasp is intuitively firm and robust, although the minimum distance  $\|\mathbf{d}(\mathbf{t})\|_{\infty} = d_{4f} \approx 0$  and consequently the measure of contact robustness is nearly zero. To obtain a less conservative estimate of how large an external disturbance can actually be resisted by the grasp, the fact that some of the contact constraints may become violated should be allowed, provided that a sufficient set of unviolated constraints remain to ensure immobilization of the

object. We explicitly note that local slippage or contact detachment are possible because of the elasticity of bodies in contact, similarly to the theory of incipient slippage in classical contact mechanics. On the other hand, such elasticity is considered as lumped in virtual springs interposed at the contacts, so that bodies still move as rigid bodies in space.

The local details of friction and elasticity at the contacts may have large influence on the phenomena occurring in grasping under the above conditions, and render an exact treatment very complex. In the following, we consider some simplifying assumptions that will allow a safe estimate of grasp robustness, with a degree of conservativeness however inferior to that of contact robustness estimates.

Our method is based on a set of simplified assumptions on the structure of the stiffness matrix  $\mathbf{K}_{i_s}$  at the  $i$ -th contact in different contact states:

- i) When both constraints (3) are fulfilled at the  $i$ -th contact, the corresponding stiffness matrix (in a local reference frame) is assumed diagonal and definite positive,  ${}^o\mathbf{K}_{i_s} = \text{diag}(K_{itx}, K_{ity}, K_{in})\mathbf{0}$ .
- ii) If the Coulomb constraint (3-b) is violated, stiffness in the tangent plane are set to zero, i.e.  ${}^o\mathbf{K}_{i_s} = \text{diag}(0, 0, K_{in})$ .
- iii) If the contact detachment condition (3-a) is violated at one contact point, the contact stiffness at that point is assumed to be null:  ${}^o\mathbf{K}_{i_s} = \mathbf{0}$ .

Clearly, such assumptions conservatively disregard the fact that locally slipping contacts continue to contribute to the force balance.

For a given grasp comprised of  $n$  contact points, let  $\mathcal{C}$  be the set of all possible combinations of the three contact states above (the cardinality of  $\mathcal{C}$  is  $3^n$ ). For each grasp configuration  $\mathcal{C}_j$  in  $\mathcal{C}$ , consider the global stiffness matrix  $\mathbf{K}(\mathcal{C}_j) = \mathbf{K}_s(\mathbf{I} - \mathbf{J}(\mathbf{J}^T \mathbf{K}_s \mathbf{J} + \mathbf{R}_q)^{-1} \mathbf{J}^T \mathbf{K}_s)$ , where  $\mathbf{K}_s = \text{diag}(\mathbf{K}_{1_s}, \dots, \mathbf{K}_{n_s})$  and local stiffness matrices are defined according to the state of the corresponding contact in  $\mathcal{C}_j$ .

A measure of potential grasp robustness can therefore be defined as

$$\max_{\mathcal{C}_j} \max_{\mathbf{y}} \frac{\|\mathbf{d}(\mathbf{G}_{K(\mathcal{C}_j)}^R \mathbf{G}\mathbf{t} + \mathbf{E}(\mathcal{C}_j)\mathbf{y})\|_{\infty}}{\sigma_{max}(\mathbf{G}_{K(\mathcal{C}_j)}^R)} \quad (11)$$

$$\text{subject to} \quad \ker(\mathbf{K}(\mathcal{C}_j)\mathbf{G}^T) = \emptyset \quad (12)$$

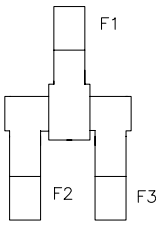


Figure 2. Tactile-force instrumented talon

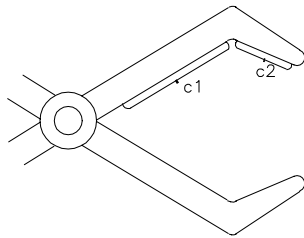


Figure 3. Talon grasping

where  $\mathbf{G}_K^R(\mathcal{C}_j)$  and  $\mathbf{E}(\mathcal{C}_j)$  are the weighted pseudoinverse (5) and the basis matrix of asymptotically reproducible internal forces (8), respectively, evaluated with  $\mathbf{K}(\mathcal{C}_j)$  modified as above described. Note that condition (12) implies that candidate grasp configurations  $\mathcal{C}_j$  need only be considered that can actually immobilize the object, thus effectively reducing the dimension of the set to be searched for the highest contact robustness.

#### 4. Experiment

Grasp analysis tools discussed in section 3 above have been employed in an experimental testbed consisting of a simple one-degree-of-freedom gripper, or “Instrumented Talon”, developed at the Artificial Intelligence Laboratory of M.I.T. for use on the M.I.T. Whole-Arm Manipulator [11]. The talon (fig.2) has three fingers each equipped with four tactile-sensitive piezoelectric pads [?], and strain-gage based force sensors at the base of the fingers. In its present version, the instrumented talon is only able to sense forces in the finger plane.

The instrumented talon shares its computational resources with the robotic system it is a part of. The complete complete computational architecture consists of five Motorola 68040 single board computers working in parallel within the HummingBird real-time software environment [?]. At present, sensory data from the talon are acquired through an interface board and two fiber optic lines to a 68040 VME. The bulk of grasp analysis computations are carried over by a second 68040 board.

In a first experiment, the talon was used to grasp a 1 Kg parallelepipedal block as depicted in fig. 4. For this experiment, the stiffness of the finger po-

sition controller was set very high. The composite grasp stiffness matrix results from (5) with  $\mathbf{R}_q \rightarrow \infty$  as  $\mathbf{K} \approx \mathbf{K}_s$ . Moreover, due to the homogeneity of materials used, the composite grasp stiffness matrix has the very simple form  $\mathbf{K} = k\mathbf{I}$ , where  $k$  is a scalar whose actual value does not influence the grasp analysis of previous sections. The friction coefficient for the considered contact conditions is ca. 0.78. For the grasp under consideration, joint angles are  $q_1 = 35\text{deg}$  and  $q_2 = -35\text{deg}$ , while the sensed contact points (in cm) and contact normals are arranged columnwise below:

$$\begin{bmatrix} -5.2 & -5.2 & 5.2 & 5.2 \\ 7.2 & 8.9 & 7.2 & 8.9 \\ 0 & 0 & 1.6 & 1.6 \end{bmatrix}; \begin{bmatrix} 0.82 & 0.99 & -0.82 & -0.99 \\ 0.57 & -0.08 & 0.57 & -0.09 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The constraint distance vector (4) evaluates for this grasp to

$$\mathbf{d}(\mathbf{t}) = [(0.1, 0.002) (0.47, 0.2) (2, 0.1) (2.8, 1.5) (2, 0.12) (2.8, 1.5)].$$

showing that contact in  $c_1$  is closest to slippage. The measure of quasi-static contact robustness (6) is in this case

$$\|\mathbf{d}(\mathbf{t})\|_{\infty} / \sigma_{max}(\mathbf{G}_K^R) = 0.002 / 26.2 = 0.000076. \quad (13)$$

In order to evaluate how much the grasp can be improved by modifying internal forces, it is necessary to determine the subspace of asymptotically reproducible internal forces. Note that, in this example, while  $\dim \ker(\mathbf{G}) = 12$ ,  $\dim \mathcal{F}_a = 1$ . In particular, contact forces in  $\mathcal{F}_a$  are depicted in fig.4.

The algorithm described in [8] for real time evaluation of the potential contact robustness measure (10) (having set  $f_{i,max} = 20\text{N}$ ) was implemented on a devoted 68040 processor, yielding a rate of 10 KHz,

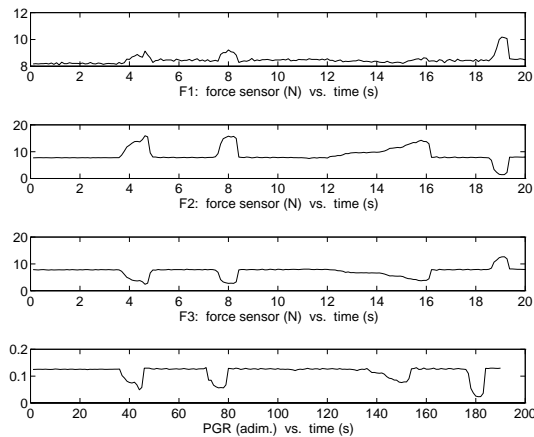


Figure 4. Force sensor outputs and estimated PGR corresponding to application of four external disturbance on the grasped object.

while sensor measurements were processed at a faster rate to estimate the external wrench  $\mathbf{w}$ . The potential contact robustness (PCR) for this grasp (obtained through averaging because of sensor noise) was  $\overline{\text{PCR}} = 0.002$ , thus showing the beneficial effect of increasing internal forces.

Finally, we consider the measure of potential grasp robustness (PGR). For the equilibrium grasp configuration of fig. 4, the (averaged) value of  $\overline{\text{PGR}} = 0.1594$  was obtained, corresponding to the grasp state when the innermost contacts on the three fingers are slipping. Note that  $\overline{\text{PGR}} \approx 80\overline{\text{PCR}}$ .

The measure of potential grasp robustness has been implemented in real time while unknown disturbances were manually applied to the object. Results are reported in fig. 4, showing the decrease of the PGR corresponding to the increase of the disturbing action on the object.

## 5. Conclusions

We have considered the robustness of robotic grasping with respect to external disturbances, in a more general framework than previous works on the same topic. Namely, we considered the case when the gripper is kinematically defective (as happens in simple

grippers and in whole-arm mechanisms), and underscored the difference between contact robustness and grasp robustness. Some preliminary experimental results have been presented, indicating the viability of the proposed tools for real-time implementation of optimizing force policies in grasping.

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