On the problem of decomposing grasp and manipulation forces in multiple whole–limb manipulation

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Abstract

By the term *power grasp* in the phisiology of human manipulation, a particular type of hold is indicated, that uses not only the fingertips but also the inner phalanges of the hand for constraining the object. In robotics, this concept can be extended to robotic systems composed of multiple actuated limbs (such as arms, fingers, or legs) cooperating in the manipulation of an object. Power grasp (also indicated by "enveloping" or "whole–limb") operations that exploit any part of the limbs to contact the object are considered in this paper. In particular, the problem of decomposing the system of contact forces exerted between the robot limbs and the object, in order to apply a desired resultant force on the object (and/or to resist external disturbances) is studied. The peculiarity of whole–limb systems is that contacts occurring on links with limited mobility, such as the inner links of a robot arm or hand, and even on fixed links (a robot chest or palm), are possible. Although the potential usefulness of whole–limb manipulation is demonstrated by biomorphic examples as well as by practically implemented robotic devices, present methods for grasp analysis cannot directly deal with these type of grasping mechanisms. We propose a modification of known force decomposition analysis that generalizes to enveloping grasping. The results of the proposed technique provide a basis for the realization of real–time optimal control of whole–limb manipulation.

1 Introduction

To afford the ever increasing level of power and flexibility demanded by sophisticated applications, robots are envisioned to evolve from the simple one–arm configuration most common today, to multiple–limb systems coordinated towards task accomplishment. As of today, several examples of such systems are already successfully operated, such as pairs of robot arms that cooperate in carrying a heavy or bulky object, robot hands that use several fingers for dextrous manipulation tasks, and deambulating vehicles that use their legs to negotiate difficult terrains and balance themselves actively. A rather extensive literature about the analysis and control of coordinated robot systems includes the study of their kinematics, statics, and dynamics. Amongst the main contributions, one can be referred to the work of Nakano *et al.* [20], Mason [15], and Uchiyama and Dauchez [24] for cooperating multiple arms; Salisbury [21], Kerr and Roth [11], Li, Hsu and Sastry [13], and Nakamura, Nagai and Yoshikawa [19] for dextrous hands; and to the work of McGhee and Orin [17], Waldron [26], and Klein and Kittivatcharapong [12] for legged vehicles.

Equipping multiple manipulation systems with the ability to use all of their links for contacting and manipulating objects is one way of further enhancing their capabilities and applicability potentials. As often occurs in robotics, this idea comes directly from the observation of human and animal examples. The arms and chest of a man carrying large objects, his hand used to firmly grasp an object between the phalanges of the fingers and the palm ("power grasping"), or the limbs of an ape when climbing a tree, provide us with the evidence of the usefulness of such "whole–limb" manipulation in nature. Trinkle [23] studied planning techniques for enveloping, frictionless grasping. Robotic devices intended to exploit the whole–limb manipulation idea have been pioneered by the MIT WAM (Whole Arm Manipulator) project, reported firstly by Salisbury [22]. A dextrous hand using all its parts (including the inner phalanges and the palm) to achieve robust power grasps and high manipulability has been proposed by Vassura and Bicchi [25]; Mirza and Orin [18] described a multiple arm manipulation system (DIGITS), and discussed the improved robustness of power grasping.

A characteristic of whole–limb manipulating systems is their use of links that have only limited mobility: for instance, the palm of a hand has no mobility at all. Thus, such systems are intrinsically defective, i.e., they possess fewer degrees–of–freedom than necessary to achieve arbitrary configurations in their operational space. It should be noted that defectivity is not a peculiarity of whole–limb systems alone. Rather, it is very common to consider systems with defective kinematics in practice. For instance, most industrial grippers are simple one– or two–degree–of–freedom devices, that cannot control arbitrary displacements of the tips of their fingers. Also, coordination of two arms of the common SCARA type is a defective kinematics problem (see [2]). Finally, non–

defective manipulation systems are a particular case of defective ones, and therefore it is desirable to obtain results in the latter, more general setting, that will be applicable to conventional robots too.

Defectivity of enveloping manipulation systems poses new problems in the analysis of kinematics, statics and dynamics, that cannot be dealt with by standard methods [3]. This paper is devoted to the investigation of a particular aspect of the analysis of defective manipulation systems, i.e. the problem of force decomposition. The basic questions we are concerned with are: a) when external forces act upon the manipulated object disturbing its equilibrium, how do they distribute between the contacts, and b) how can we modify the contact forces so as to achieve desirable values in spite of external disturbances. Note that the problem of controlling the distribution of contact forces in a manipulation system such as a hand, a pair of cooperating robot arms, or a legged vehicle, is of crucial importance for the realization of most basic control tasks, such as coordinated motion, internal force control, grasp stabilization etc. The scope of this paper is limited to giving a quasi–static analysis of force distribution, that concerns steady–state solutions to the above problems. The quasi–static approach allows a very simple understanding of the geometry of the problem, and results are obtained that provide a basis for building suitable control laws to implement specific grasping tasks (note that dynamics do not typically play a major role in the control of grasping, where motions are usually slow). The dynamic analysis of this problems is not feasible in general without resorting to linearization of the complex robot–object model. In a linearized setting, results of the analysis of dynamic structural properties reported in [4] show that what is called "active internal forces" in the context of this paper, can be described as "functionally controllable outputs at steady–state" in a properly defined dynamic system.

2 Background

The problem of controlling contact forces in a multiple manipulation system such as a hand, a pair of cooperating robot arms, or a legged vehicle, has been traditionally considered in the assumption that every single finger (arm, or leg: in the following, we will refer to "hands" generically) has full mobility in its task space. This assumption greatly simplifies the problem, by allowing to separately deal with the analysis of the distribution of grasp force among the contacts, and with the control of the joint torques that realize desirable contact forces. In this section we briefly review the background on grasp analysis techniques, in order to highlight what new problems are posed by defective systems.

Let for instance an object be grasped by means of n contacts and let the components of contact forces and moments on the object form a vector[1] $\mathbf{t} \in \mathbb{R}^t$. Consider the task of resisting an external force $\mathbf{f} \in \mathbb{R}^3$ and moment $\mathbf{m} \in \mathbb{R}^3$ applied upon the object (the task of steering an object along a desired trajectory is equivalent once the inertial load corresponding to the specified acceleration and velocity profile is determined). The force and moment balance equation for the object can be written in matrix notation as

$$\mathbf{w} = -\mathbf{G}\mathbf{t}, \tag{1}$$

where $\mathbf{w} = (\mathbf{f}^{\mathrm{T}}\,\mathbf{m}^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^6$ is the so–called load "wrench", and $\mathbf{G} \in \mathbb{R}^{6\times t}$ is usually termed as the "grasp matrix", or "grip transform". This equation has a solution in the hypothesis that $\mathbf{w}$ belongs to the range space of $\mathbf{G}$, (i.e., $\mathbf{w} \in \mathcal{R}(\mathbf{G})$). In the following we assume, unless otherwise stated, that $\mathcal{R}(\mathbf{G}) = \mathbb{R}^6$, so that the existence of a solution to (1) for any $\mathbf{w}$ is guaranteed. In general, (1) has more unknowns than equations, so that the solution is not unique. The general solution can be written as

$$\mathbf{t} = -\mathbf{G}^R\,\mathbf{w} + \mathbf{A}\mathbf{x}; \tag{2}$$

where $\mathbf{G}^R$ is a right–inverse of the grasp matrix, and $\mathbf{A} \in \mathbb{R}^{t\times h}$ is a matrix whose columns form a basis of the nullspace of $\mathbf{G}$ (noted with $\mathcal{N}(\mathbf{G})$). The coefficient vector $\mathbf{x} \in \mathbb{R}^h$ parameterizes the homogeneous part of the solution (2): for any choice of $\mathbf{x}$, a vector of contact forces results that equilibrates the desired load. Most known grasp optimization techniques (see e.g. [19]), can be formulated by defining a cost function $V(\mathbf{x})$ and constraint functions $g_i(\mathbf{x})$ as

Find $\hat{\mathbf{x}}$ *such that*
$V(\hat{\mathbf{x}}, \mathbf{w})$ *is minimum;*
$g_i(\hat{\mathbf{x}}) \leq 0$.

The cost and constraint functions usually are designed so as to realize the goals of avoiding contact slippage and minimizing consumption of power in the joint actuators. Standard non–linear programming algorithms are available to find $\hat{\mathbf{x}}$. The corresponding $\hat{\mathbf{t}} = -\mathbf{G}^R\mathbf{w} + \mathbf{A}\hat{\mathbf{x}}$ is the optimal force distribution among contacts with respect to the criterion adopted in the design of V. *Finally,* $\hat{\mathbf{t}}$ *is applied by the fingers under some type of force control technique.*

It should be noted that this last sentence tacitly relies upon a fundamental underlying assumption, i.e. that any arbitrary distribution of contact forces $\mathbf{t}$ can be actively controlled by the robot. To discuss this assumption, consider the linear relationship between the contact forces on the fingers and the vector $\tau \in \mathbb{R}^q$ of joint actuator torques:

$$\tau = \mathbf{J}^{\mathrm{T}}\,\mathbf{t}. \tag{3}$$

[1] Notation will be more precisely defined in section 3

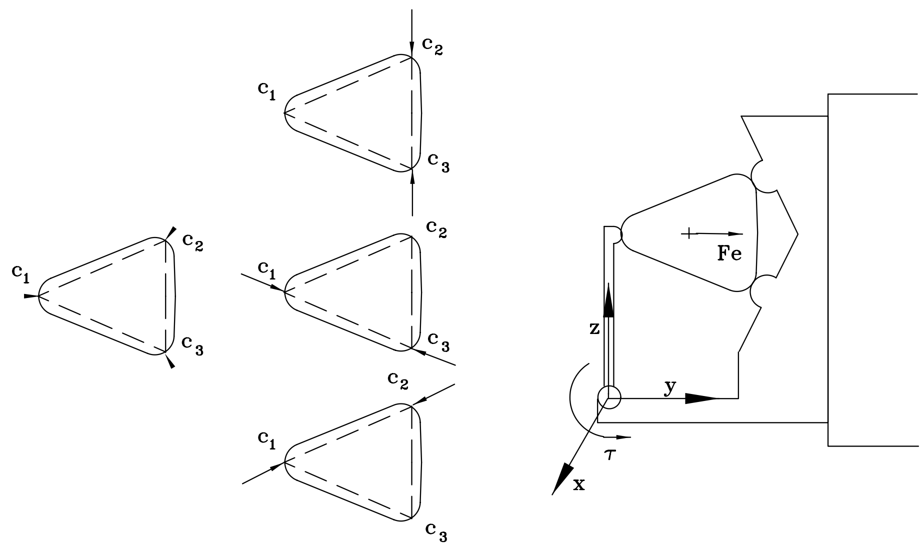
where the matrix $\mathbf{J} \in \mathbb{R}^{t \times q}$ is the "Jacobian" of the manipulation system. A robot system with $q >$ rank $(\mathbf{J}) = t$ is a "redundant" system, while if $q \geq$ rank $(\mathbf{J}) < t$, the robot system is "defective" with respect to its task space dimension. For a redundant manipulator, a many–to–one mapping $\mathbf{t}(\tau) : \mathbb{R}^q \to \mathbb{R}^t$ can always be established which is onto $\mathbb{R}^t$. For a non–redundant, non–defective (minimal) manipulator with $q =$ rank $(\mathbf{J}) = t$, a one–to–one and onto mapping can be established between $\mathbb{R}^q$ and $\mathbb{R}^t$. In both cases, arbitrary $\mathbf{t}$'s can be realized by suitably regulating joint torques.

However, solution 2 can not in general be applied to defective manipulating systems, since there is no guarantee that the optimal contact forces can actually be realized by the robot. In other words, complete (output function) controllability of internal forces may not be achieved in those cases. While the controllability concept can be investigated in a dynamical model of grasping, in this paper we undertake a quasi–static analysis meant to answer the question, "what internal forces at equilibrium are modifiable at will, when inputs are joint torques?".

Consider for example the grasp of the object depicted in fig.1–a by means of three contacts placed in $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$. Intuitively, there are three possible independent combinations of contact forces giving homogeneous solutions to the grasp equations, namely those pushing or pulling the object along the edges of the so–called "grasp triangle" (fig.1–b). Any pair of these "internal" forces or their combinations may be used, for instance, to squeeze the object and decrease the danger of slippage. However, if the grasp is to be realized by the simple single–joint gripper shown in fig.1–c, it appears that some configuration of internal forces may not be feasible (for instance, opposing forces in the direction $\mathbf{c}_2 - -\mathbf{c}_3$ as shown in the uppermost part of fig.1–b). In order to solve the force decomposition problem for general manipulation systems, a more accurate analysis is therefore necessary, which takes into account the kinematics and the deformability of the manipulation system.

3 Quasi–Static Model of Whole–Limb Manipulation

The model of the cooperating manipulation system we assume is comprised of an arbitrary number of robot "fingers" (i.e., simple chains of links connected through revolute or prismatic joints), and of an object, which is in contact with some or all of the links (see fig.2). We assume that, for the i–th of the n contacts, the location of the contact point $\mathbf{c}_i \in \mathbb{R}^3$ is known, by either planning or sensing. According to standard conventions, we consider a fixed "base" reference frame B, and local reference frames E_j, fixed to the j–th robot link (see fig.2): the position of the origin of E_j is placed on the j–th joint axis, and is designed in base frame by the 3–vector $\mathbf{o}_j$. The z–axis of E_j is aligned with the



a) b) c)

Figure 1: A simple example of grasp with a kinematically defective device.

j–th joint axis, and has unit vector $\mathbf{z}_j$ in base frame; the x–axis of E_j is aligned with the line joining $\mathbf{o}_j$ with $\mathbf{o}_{j+1}$. Frames associated with fixed and extremal links can be chosen with some degree of arbitrarity. All vectors are expressed in base frame, unless explicitly noted. Let the contact force and torque exerted on the object at the i–th contact be $\mathbf{p}_i \in \mathbb{R}^3$ and $\mathbf{m}_i \in \mathbb{R}^3$, respectively, and put $\tilde{\mathbf{t}} = (\mathbf{p}_1^{\mathrm{T}}, \ldots, \mathbf{p}_n^{\mathrm{T}}, \mathbf{m}_1^{\mathrm{T}}, \ldots, \mathbf{m}_n^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^{6n}$. Balance equation for the object can be written in matrix form as

$$\mathbf{w} = -\tilde{\mathbf{G}}\tilde{\mathbf{t}}, \tag{4}$$

where

$$\tilde{\mathbf{G}} = \left(\begin{array}{cccc|cccc} \mathbf{I}_3 & \mathbf{I}_3 & \cdots & \mathbf{I}_3 & & \mathbf{O}_{3\times 3n} & & \\ \mathbf{S}(\mathbf{c_1}) & \mathbf{S}(\mathbf{c_2}) & \cdots & \mathbf{S}(\mathbf{c_n}) & \mathbf{I}_3 & \mathbf{I}_3 & \cdots & \mathbf{I}_3 \end{array} \right);$$

and $\mathbf{S}(\mathbf{c_i})$ is the cross–product matrix for $\mathbf{c}_i$ (i.e. the skew–symmetric matrix such that $\mathbf{S}(\mathbf{c_i})\mathbf{p}_i = \mathbf{c}_i \times \mathbf{p}_i$). Analogously, force balance equations for the manipulator joints can be written as

$$\tau = \tilde{\mathbf{J}}^{\mathrm{T}}\,\tilde{\mathbf{t}}, \tag{5}$$

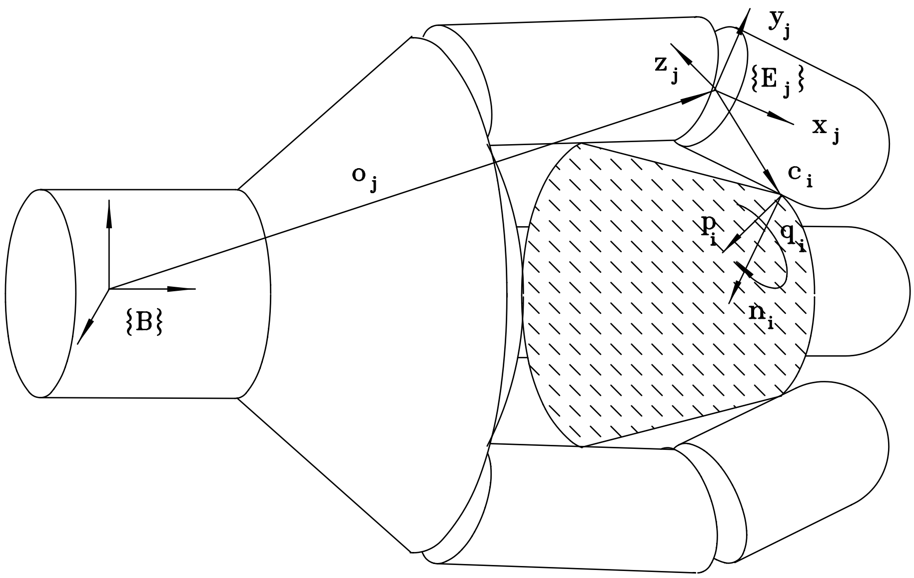


Figure 2: Local and base reference frames in whole–limb manipulation.

where $\tilde{\mathbf{J}}$ is a $6n \times q$ matrix whose elements are functions of the robot geometric parameters and joint angles:

$$\tilde{\mathbf{J}}^{\mathrm{T}} = \left(\begin{array}{cccc|cccc} \mathbf{D}_{1,1} & \mathbf{D}_{2,1} & \cdots & \mathbf{D}_{n,1} & \mathbf{L}_{1,1} & \mathbf{L}_{2,1} & \cdots & \mathbf{L}_{n,1} \\ \mathbf{D}_{1,2} & \mathbf{D}_{2,2} & \cdots & \mathbf{D}_{n,2} & \mathbf{L}_{1,2} & \mathbf{L}_{2,2} & \cdots & \mathbf{L}_{n,2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{D}_{1,q} & \mathbf{D}_{2,q} & \cdots & \mathbf{D}_{n,q} & \mathbf{L}_{1,q} & \mathbf{L}_{2,q} & \cdots & \mathbf{L}_{n,q} \end{array} \right), \tag{6}$$

and the 1×3 blocks $\mathbf{D}_{i,j}$ and $\mathbf{L}_{i,j}$ are defined as

$$\mathbf{D}_{i,j} = \begin{cases} \mathbf{O}_{1\times 3} & \text{if the } i\text{–th contact force does not affect the } j\text{–th joint;} \\ \mathbf{z}_j^{\mathrm{T}} & \text{for prismatic } j\text{–th joint;} \\ \mathbf{z}_j^{\mathrm{T}} \; \mathbf{S}(\mathbf{c}_i - \mathbf{o}_j) & \text{for revolute } i\text{–th joint;} \end{cases}$$

$$\mathbf{L}_{i,j} = \begin{cases} \mathbf{O}_{1\times 3} & \text{if the } i\text{–th contact torque does not affect the } j\text{–th joint;} \\ \mathbf{O}_{1\times 3} & \text{for prismatic } j\text{–th joint;} \\ \mathbf{z}_j^{\mathrm{T}} & \text{for revolute } j\text{–th joint;} \end{cases}$$

To incorporate contact constraints in the model, relative displacements between the object and the links at the contact points must be considered. Therefore, we introduce n reference frames ${}^{o}C_i$ fixed w.r.t. the object and centered in $\mathbf{c}_i$; and n reference frames ${}^{m}C_i$, each fixed w.r.t. the link that touches the object in $\mathbf{c}_i$, and centered in $\mathbf{c}_i$. Corresponding to a small displacement $\Delta\mathbf{r}$ and rotation $\Delta\phi$ of the object w.r.t. the base frame (summarized in the "twist" vector $\Delta \mathbf{u} = (\Delta \mathbf{r}^{\mathrm{T}}, \Delta \phi^{\mathrm{T}})^{\mathrm{T}}$), frames ${}^{o}C_i$ undergo a displacement $\Delta^{o}\mathbf{c}_i$ and rotation $\Delta^{o}\phi_i$ whose relationship with $\Delta \mathbf{u}$ can be derived by the virtual work principle as

$$\Delta^{o}\mathbf{x} = \tilde{\mathbf{G}}^{\mathrm{T}} \ \Delta \mathbf{u}, \tag{7}$$

$$\Delta^{o}\mathbf{x} = (\Delta^{o}\mathbf{c}_1^{\mathrm{T}}, \ldots, \Delta^{o}\mathbf{c}_n^{\mathrm{T}}, \Delta^{o}\phi_1^{\mathrm{T}}, \ldots, \Delta^{o}\phi_n^{\mathrm{T}})^{\mathrm{T}}.$$

Analogous is the relationship between small displacements of the joints $\Delta \mathbf{q}$ and the displacements $\Delta^{m}\mathbf{c}_i$ and rotations ${}^{m}\Delta\phi_i$ of the contact frames ${}^{m}C_i$:

$$\Delta^{m}\mathbf{x} = \tilde{\mathbf{J}} \ \Delta \mathbf{q}, \tag{8}$$

$$\Delta^{m}\mathbf{x} = (\Delta^{m}\mathbf{c}_1^{\mathrm{T}}, \ldots, \Delta^{m}\mathbf{c}_n^{\mathrm{T}}, \Delta^{m}\phi_1^{\mathrm{T}}, \ldots, \Delta^{m}\phi_n^{\mathrm{T}})^{\mathrm{T}}.$$

Contact constraints impose that certain components of the relative displacements $\Delta^{o}\mathbf{x} - \Delta^{m}\mathbf{x}$ are selectively opposed by reaction forces, depending upon the type of contact. Several types of contact models can be used to describe the interaction between the links and the object, among which the most useful are probably the point–contact–with–friction (also called "hard–finger") model, the "soft–finger" model, and the complete–constraint (or "very–soft–finger") model. For a description of these models, see e.g. [21]. Contact constraints can be expressed in terms of a suitable selection matrix $\mathbf{H}$: assuming that the contacts are numbered according to their type, so that the variables relative to complete–constraint contacts have indices from 1 to l, those relative to soft–finger contacts from $l+1$ to $l+m$, and those relative to hard–finger contacts from $l+m+1$ to $l+m+r=n$, the kinematic constraint for a rigid–body manipulation system can be expressed as

$$\begin{aligned} \mathbf{H}(\Delta^{m}\mathbf{x} - \Delta^{o}\mathbf{x}) &= 0, \qquad (9) \\ \mathbf{H} &= \left(\begin{array}{c|ccc|c} \mathbf{I}_{3(n+l)} & \multicolumn{4}{c}{\mathbf{O}_{3(n+l)\times 3(n-l)}} \\ \hline & \mathbf{n}_{l+1}^{\mathrm{T}} & \cdots & 0\ 0\ 0 & \\ \mathbf{O}_{m\times 3(n+l)} & \cdots & \cdots & \cdots & \mathbf{O}_{m\times 3r} \\ & 0\ 0\ 0 & \cdots & \mathbf{n}_{l+m}^{\mathrm{T}} & \end{array} \right), \end{aligned}$$

where $\mathbf{n}_i$ is the unit normal vector to the contacting surfaces at the i–th contact point. The selection matrix $\mathbf{H}$ has $6n$ columns and $3n + 3l + m = t$ rows.

All relationship considered so far are valid for a rigid–body model of the robot system. However, the force distribution problem for general systems is underdetermined. To solve the indeterminacy, the rigid body model is inadequate, and a more accurate model, taking into account the elastic elements that are involved in the system, has therefore to be considered[2]. This can be conceptually done by introducing a set of "virtual springs"

[2]since in this paper only quasi–static grasps are considered, dynamic effects such as viscous damping, are disregarded in the model

interposed between the links and the object at the contact points, such that the elastic relationship between the relevant components of the relative displacements $\Delta^o\mathbf{x} - \Delta^m\mathbf{x}$ and the corresponding components of contact forces can be written as

$$\mathbf{t} = \mathbf{K}\mathbf{H}(\Delta^m\mathbf{x} - \Delta^o\mathbf{x}) + \mathbf{t}_o, \tag{10}$$

where $\mathbf{t}_o$ is the contact force in the reference configuration $\Delta^m\mathbf{x} = \Delta^o\mathbf{x} = 0$. Eq.(10) establishes a relationship between certain components of contact forces and relative displacements at the contacts, according to different types of contact. In (10), $\mathbf{t} \in \mathbb{R}^t$ differs from $\tilde{\mathbf{t}} = \mathbf{H}^{\mathrm{T}}\mathbf{t} \in \mathbb{R}^{6n}$ in that only the components of contact force and torque which are relevant to contact description are considered. The stiffness matrix $\mathbf{K} \in \mathbb{R}^{t\times t}$ incorporates the structural elasticity of the object and of the fingers, and the stiffness of joint servos if position controllers are used [16]. Denoting with $\mathbf{C}_s$ the $t \times t$ structural compliance matrix (due e.g. to the flexibility of links and mechanical transmission, or to soft gripping surfaces), and with $\mathbf{C}_q$ the $q \times q$ the diagonal matrix whose element in position i, i is the inverse of the steady–state gain of the i–th joint position servo, we have

$$\mathbf{K} = (\mathbf{C}_s + \mathbf{J}\mathbf{C}_q\mathbf{J}^{\mathrm{T}})^{-1}. \tag{11}$$

As a consequence of its physical nature, $\mathbf{K}$ can be assumed non–singular. A detailed and comprehensive study on the evaluation and the realization of desirable stiffness matrices with articulated hands has been presented by Cutkosky and Kao [6]. It should be noted that, since $\mathbf{K}$ includes the stiffness of the joint position controllers, the displacement vector $\Delta\mathbf{q}$ must be interpreted as the change in the input reference for position controllers.

In view of the above definitions, the matrices $\mathbf{G}$ and $\mathbf{J}$ introduced in (1) and (3), are evaluated as

$$\begin{aligned} \mathbf{G} &= \tilde{\mathbf{G}}\mathbf{H}^{\mathrm{T}}; \\ \mathbf{J} &= \mathbf{H}\tilde{\mathbf{J}}, \end{aligned}$$

and the quasi–static model of the manipulation system to be studied can be summarized by the following equations:

$$\mathbf{w} = -\mathbf{G}\mathbf{t}; \tag{12}$$

$$\tau = \mathbf{J}^{\mathrm{T}}\mathbf{t}; \tag{13}$$

$$\mathbf{t} = \mathbf{K}(\mathbf{J}\Delta\mathbf{q} - \mathbf{G}^{\mathrm{T}}\Delta\mathbf{u}) + \mathbf{t}_o. \tag{14}$$

4 The Particular Solution

The particular solution $\mathbf{t}_p = -\mathbf{G}^R\mathbf{w}$ of the force distribution problem (1) is not unique, since $\mathbf{G}$ in general admits infinitely many right inverses. However, we expect a unique solution to the following

Force distribution problem. Assume that an object, at equilibrium under an external load $\mathbf{w}_o$ and contact forces $\mathbf{t}_o$, is subject to an additional load $\mathbf{w}$, while all other parameters are kept constant. Determine the values of contact forces at the new equilibrium.

Proposition 1 *The solution to the force distribution problem is unique, and is given by*

$$\mathbf{t} = \mathbf{G}_K^R \mathbf{w} + \mathbf{t}_o, \tag{15}$$

where $\mathbf{G}_K^R = \mathbf{K}\mathbf{G}^{\mathrm{T}}(\mathbf{G}\mathbf{K}\mathbf{G}^{\mathrm{T}})^{-1}$.

Proof. Let $\Delta\mathbf{q}$ and $\Delta\mathbf{u}$ be the displacements of the joints and of the object at the new equilibrium reached under $\mathbf{w}_o + \mathbf{w}$, w.r.t. the original equilibrium configuration. Since the reference position of joint controllers are kept constant, $\Delta\mathbf{q} = 0$. Substituting (14) in (12), we have

$$\begin{aligned} \mathbf{t} &= -\mathbf{K}\mathbf{G}^{\mathrm{T}}\Delta\mathbf{u} + \mathbf{t}_o; && (16)\\ \mathbf{w} + \mathbf{w}_o &= \mathbf{G}\mathbf{K}\mathbf{G}^{\mathrm{T}}\Delta\mathbf{u} - \mathbf{G}\mathbf{t}_o, && (17) \end{aligned}$$

Hence, recalling that $\mathbf{G}$ is assumed full row rank and $\mathbf{K}$ is invertible,

$$\mathbf{t} = -\mathbf{K}\mathbf{G}^{\mathrm{T}}(\mathbf{G}\mathbf{K}\mathbf{G}^{\mathrm{T}})^{-1}\mathbf{w} + \mathbf{t}_o = \mathbf{G}_K^R\mathbf{w} + \mathbf{t}_o.\square \tag{18}$$

It can be observed that $\mathbf{G}_K^R$ is the $\mathbf{K}$–weighted pseudoinverse of $\mathbf{G}$, providing the particular solution that minimizes the elastic energy $1/2(\Delta^m\mathbf{x} - \Delta^o\mathbf{x})^{\mathrm{T}}\mathbf{K}(\Delta^m\mathbf{x} - \Delta^o\mathbf{x})$ (see e.g. [9], [10]).

Among the infinite possible numerical right–inverses of $\mathbf{G}$, $\mathbf{G}_K^R$ appears to be phisically well motivated. The importance of phisically sound bases in the choice of numerical algorithms have been shown in several recent papers in the robotics literature, e.g. [7], [8]. In view of such remarks, it is of interest to show the following

Proposition 2 *The particular solution (15) is invariant under linear non–singular transformations of the variables.*

Proof. Suppose that the representation of the variables is changed (due to changes of reference frame or measurement units, for instance) as

$$\begin{aligned} \bar{\mathbf{w}} &= \mathbf{T}_w\mathbf{w}; & \Delta\bar{\mathbf{u}} &= \mathbf{T}_u\Delta\mathbf{u};\\ \bar{\mathbf{t}} &= \mathbf{T}_t\mathbf{t}; & \Delta\bar{\mathbf{x}} &= \mathbf{T}_x\Delta\mathbf{x}. \end{aligned}$$

Note that $\mathbf{T}_w$ and $\mathbf{T}_t$ may be completely arbitrary non–singular matrices. For the sake of preserving the physical consistency of the representations, however, constraints must be imposed on the choice of $\mathbf{T}_u$ and $\mathbf{T}_x$: in fact, the requirement that $\mathbf{w}^{\mathrm{T}}\,\Delta\mathbf{u} = \bar{\mathbf{w}}^{\mathrm{T}}\,\Delta\bar{\mathbf{u}}$ entails $\mathbf{T}_u^{\mathrm{T}} = \mathbf{T}_w^{-1}$, and $\mathbf{T}_x^{\mathrm{T}} = \mathbf{T}_t^{-1}$. The transformed form of (12) and (16) can be obtained as

$$\bar{\mathbf{w}} = -\bar{\mathbf{G}}\bar{\mathbf{t}}; \quad \bar{\mathbf{t}} = -\bar{\mathbf{K}}\bar{\mathbf{G}}^{\mathrm{T}}\,\Delta\bar{\mathbf{u}},$$

where

$$\bar{\mathbf{G}} = \mathbf{T}_w\mathbf{G}\mathbf{T}_t^{-1}; \quad \bar{\mathbf{K}} = \mathbf{T}_t\mathbf{K}\mathbf{T}_x^{-1}.$$

Eq.(18) can be rewritten in terms of the transformed variables (assuming $\mathbf{t}_o = 0$ for simplicity) as

$$\bar{\mathbf{t}} = -\bar{\mathbf{G}}_K^R\bar{\mathbf{w}} = -\bar{\mathbf{K}}\bar{\mathbf{G}}^{\mathrm{T}}\,(\bar{\mathbf{G}}\bar{\mathbf{K}}\bar{\mathbf{G}}^{\mathrm{T}}\,)^{-1}\bar{\mathbf{w}},$$

It can be easily verified by substitution that

$$\bar{\mathbf{t}} = -\bar{\mathbf{G}}_K^R\bar{\mathbf{w}} = -\mathbf{T}_t\mathbf{G}_K^R\mathbf{T}_w^{-1}\bar{\mathbf{w}},$$

i.e., the particular solution (18) evaluated after a variable transformation is equal to the transformation of the solution obtained prior to transformation.□

Note that the invariancy property of (18) is not shared by other commonly used particular solutions. For instance, by applying the straightforward pseudo–inverse solution

$$\mathbf{t}_p = \mathbf{G}^{+}\mathbf{w}, \quad \mathbf{G}^{+} = \mathbf{G}^{\mathrm{T}}\,(\mathbf{G}\mathbf{G}^{\mathrm{T}}\,)^{-1}, \tag{19}$$

to a transformed representation of the problem, we obtain

$$\bar{\mathbf{t}}_p \;=\; \mathbf{T}_t^{-T}\mathbf{G}^{\mathrm{T}}\,\mathbf{T}_w^{\mathrm{T}}\,(\mathbf{T}_w\mathbf{G}\mathbf{T}_t^{-1}\mathbf{T}_t^{-T}\mathbf{G}^{\mathrm{T}}\,\mathbf{T}_w^{\mathrm{T}}\,)^{-1}\;\bar{\mathbf{w}} \;=\; \mathbf{T}_t^{-T}\mathbf{G}^{\mathrm{T}}\,(\mathbf{G}\mathbf{T}_x^{\mathrm{T}}\,\mathbf{T}_x\mathbf{G}^{\mathrm{T}}\,)^{-1}\mathbf{T}_w^{-1}\;\bar{\mathbf{w}}, \tag{20}$$

which differs from $\bar{\mathbf{t}}_p = \mathbf{T}_t\mathbf{G}^{+}\mathbf{T}_w^{-1}\bar{\mathbf{w}}$ whenever the transformation $\mathbf{T}_x$ (and hence $\mathbf{T}_t$) is not orthogonal.

5 The homogeneous solution

Internal forces, i.e. self–balanced contact forces that have no effect on the global motion of the manipulated object but significantly affect the grasp stability, have been identified with homogeneous solutions of (12). In mathematical terms, internal forces are elements of the subspace $\mathcal{F}_h \in \mathbb{R}^t = \mathcal{N}(\mathbf{G})$, and, by definition of $\mathbf{A}$, $\mathcal{F}_h \in \mathbb{R}^t = \mathcal{R}(\mathbf{A})$. However, as discussed above for general manipulation systems, not all homogeneous solutions may be actively controlled by using joint variables as inputs. Internal contact forces that are not actively realizable through joint control may still be present in a system, due to its initial conditions — e.g., they may have been set by prestressing elastic elements in the manipulation system. In this section we propose a decomposition of the homogeneous subspace in a subspace $\mathcal{F}_{hr}$ of active, internal contact forces and a subspace $\mathcal{F}_{ho}$ of passive (preload), internal contact forces.

5.1 Active Internal Forces

Consider an equilibrium configuration of the manipulation system under an external load $\mathbf{w}_o$, and denote with $\mathbf{q}_o$ and with $\mathbf{t}_o$ the joint positions and the contact forces in such reference configuration, respectively. By modifying the joint reference position by $\Delta\mathbf{q}$, the equilibrium configuration of the object, still subject to $\mathbf{w}_o$, is changed by $\Delta\mathbf{u}$. Correspondingly, contact forces are $\mathbf{t} = \mathbf{t}_o + \Delta\mathbf{t}$. From $\mathbf{G}\mathbf{t} = \mathbf{w}_o$ follows that $\Delta\mathbf{t} \in \mathcal{N}(\mathbf{G})$. We define *active* those internal contact forces $\Delta\mathbf{t}$ that correspond to controllable modifications of the system configuration, and let $\mathcal{F}_{hr} \in \mathbb{R}^t$ denote the set of all active $\Delta\mathbf{t}$'s.

In order to characterize the set $\mathcal{F}_{hr}$, we will make use of the principle of virtual work (P.V.W.), in the form that applies to mechanical systems constrained by bilateral, frictionless and/or rolling constraints, which is recalled here:

> **Principle of Virtual Work.** A mechanical system is in equilibrium under external forces and constraints if and only if the work of external forces corresponding to any virtual displacement of the system is zero.

Note that the P.V.W. can be applied to the general system modeled in (12) – (14) insofar as there is no slippage nor discontinuity of contacts between the object and the links. This condition can be guaranteed by applying a suitable grasp force control policy, such as discussed in the references cited above (see also below section 6).

Proposition 3 *The set of active internal forces $\mathcal{F}_{hr}$ is a linear subspace of $\mathbb{R}^t$, i.e., every active internal force can be written as the product of a basis matrix $\mathbf{E}$ times an arbitrary coefficient vector $\mathbf{y}$ of suitable dimension.*

Proof. Consider a system in the equilibrium configuration described by $\mathbf{w}_o$, $\mathbf{q}_o$, $\mathbf{t}_o$, and let $\delta\mathbf{u}$ be a displacement of the object which is compatible with all the constraints imposed by contacts with the robot links (i.e., $\delta\mathbf{u}$ is a virtual displacement of the object). Applying the P.V.W. and (12), we have immediately

$$\mathbf{w}_o^{\mathrm{T}}\,\delta\mathbf{u} = \mathbf{t}_o^{\mathrm{T}}\,\mathbf{G}^{\mathrm{T}}\,\delta\mathbf{u} = 0, \quad \forall\delta\mathbf{u}.$$

By imposing joint displacements $\Delta\mathbf{q}$, the equilibrium configuration is perturbed. A new equilibrium under the same external force $\mathbf{w}_o$ will be reached on condition that the P.V.W. is satisfied:

$$\mathbf{w}_o^{\mathrm{T}}\,\delta\mathbf{u} = (\mathbf{t}_o + \Delta\mathbf{t})^{\mathrm{T}}\,\mathbf{G}^{\mathrm{T}}\,\delta\mathbf{u} = \Delta\mathbf{t}^{\mathrm{T}}\,\mathbf{G}^{\mathrm{T}}\,\delta\mathbf{u} = 0, \quad \forall\delta\mathbf{u}.$$

From (14), $\Delta\mathbf{t} = \mathbf{K}(\mathbf{J}\Delta\mathbf{q} - \mathbf{G}^{\mathrm{T}}\,\Delta\mathbf{u})$. After substitution, the P.V.W. condition is

$$\Delta\mathbf{q}^{\mathrm{T}}\,\mathbf{J}^{\mathrm{T}}\,\mathbf{K}^{\mathrm{T}}\,\mathbf{G}^{\mathrm{T}}\,\delta\mathbf{u} = \Delta\mathbf{u}^{\mathrm{T}}\,\mathbf{G}\mathbf{K}^{\mathrm{T}}\,\mathbf{G}\delta\mathbf{u} \;\; \forall\delta\mathbf{u}$$

which implies

$$\mathbf{GKJ}\Delta\mathbf{q} = \mathbf{GKG}^{\mathrm{T}}\,\Delta\mathbf{u}, \tag{21}$$

and hence

$$\begin{aligned}\Delta\mathbf{u} &= (\mathbf{GKG}^{\mathrm{T}})^{-1}\mathbf{GKJ}\Delta\mathbf{q} && (22)\\ \Delta\mathbf{t} &= \mathbf{K}(\mathbf{J}\Delta\mathbf{q} - \mathbf{G}^{\mathrm{T}}(\mathbf{GKG}^{\mathrm{T}})^{-1}\mathbf{GKJ}\Delta\mathbf{q}) = \\ &= (\mathbf{I} - \mathbf{G}_K^R\mathbf{G})\mathbf{KJ}\Delta\mathbf{q} && (23)\end{aligned}$$

Therefore, all active internal forces can be expressed as

$$\tilde{\mathbf{t}} = \mathbf{E}\mathbf{y}, \tag{24}$$

where the columns of the $t \times e$ matrix $\mathbf{E}$ form a basis of the range of $(\mathbf{I} - \mathbf{G}_K^R\mathbf{G})\mathbf{KJ}$. □

The vector $\mathbf{y} \in \mathbb{R}^e$ is comprised of e free variables among which the $\hat{\mathbf{y}}$ corresponding to an optimal grasp force distribution can be chosen by means of suitable cost functions and optimization routines. Note that in general $e \leq h$, i.e. active internal forces are "fewer" than internal forces, corresponding to intuition. Also, $e \leq q$, i.e. no more independent active internal forces can be controlled than are joints in the system.

From a computational point of view, the algorithm sketched in the proof of Proposition 3 to evaluate the desired basis matrix $\mathbf{E}$ is not optimal, since it entails the explicit calculation of the right–inverse $\mathbf{G}_K^R$. A more efficient algorithm, which also provides further insight in the problem, can be derived by rewriting (21) as

$$\mathbf{G}\,(\mathbf{KJ}\,\Delta\mathbf{q}\; - \mathbf{KG}^{\mathrm{T}}\,\Delta\mathbf{u}) = 0,$$

or, equivalently, as

$$\mathbf{Ax} = \mathbf{KJ}\Delta\mathbf{q} - \mathbf{KG}^{\mathrm{T}}\,\Delta\mathbf{u}.$$

This equation can be recast in block matrix form as

$$\begin{bmatrix}\mathbf{A} & -\mathbf{KJ} & \mathbf{KG}^{\mathrm{T}}\end{bmatrix}\begin{pmatrix}\mathbf{x}\\ \Delta\mathbf{q}\\ \Delta\mathbf{u}\end{pmatrix} = 0. \tag{25}$$

Put $\mathbf{Q} = \begin{bmatrix}\mathbf{A} & -\mathbf{KJ} & \mathbf{KG}^{\mathrm{T}}\end{bmatrix} \in \mathbb{R}^{t\times(h+q+6)}$, and let $\mathbf{B} \in \mathbb{R}^{(h+q+6)\times b}$ be a matrix whose columns span the nullspace of $\mathbf{Q}$ (whose nullity is b). Finally, partition $\mathbf{B}$ as $\mathbf{B} = \begin{bmatrix}\mathbf{B}_1^{\mathrm{T}} & \mathbf{B}_2^{\mathrm{T}} & \mathbf{B}_3^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$, where $\mathbf{B}_1 \in \mathbb{R}^{h\times b}$, $\mathbf{B}_2 \in \mathbb{R}^{q\times b}$, and $\mathbf{B}_3 \in \mathbb{R}^{6\times b}$. The subspace under investigation is thus obtained as

$$\mathcal{F}_{hr} = \mathcal{R}(\mathbf{AB}_1).$$

and the matrix $\mathbf{E}$ is obtained by using only the independent columns of $\mathbf{AB}_1$. This method, though seemingly complex, is numerically more efficient and robust than the previously presented, since it avoids any matrix inversion. As a consequence, the latter algorithm can be applied to cases when $\mathbf{G}$ is not full row rank. Further, by using (25) it can be easily calculated which joints displacements must be commanded if a desired active internal force $\Delta\hat{\mathbf{t}} = \mathbf{E}\hat{\mathbf{y}}$ is to be applied:

$$\Delta\hat{\mathbf{q}} = \mathbf{B}_2(\mathbf{AB}_1)^{+}\mathbf{E}\hat{\mathbf{y}} \tag{26}$$

The equilibrium position of the object is correspondingly displaced by

$$\Delta\hat{\mathbf{u}} = \mathbf{B}_3(\mathbf{AB}_1)^{+}\mathbf{E}\hat{\mathbf{y}}.$$

5.2 Preload internal forces

As mentioned above, in general manipulation systems there may be internal contact forces that can not be actively controlled by means of joint displacements. Therefore, these forces will remain constantly equal to their initial value (or evolve freely, if considered in a dynamic setting). In a mechanical jig, such forces can be set once and for all as a preload condition, for instance for preventing slippage. Although in robotic systems it may be unlikely to encounter such preload forces, their analysis is an interesting completion to the study above.

Let $\mathcal{F}_{ho} \in \mathbb{R}^t$ denote the subspace of internal, passive (preload) contact forces, and let the subspace of contact forces that the manipulation system can exert on the object with zero joint torques be

$$\mathcal{F}_o = \{\mathbf{t} \in \mathbb{R}^t |\ \mathbf{J}^{\mathrm{T}}\,\mathbf{t} = 0\} \equiv \{\mathbf{t} \in \mathbb{R}^t |\ \mathbf{t} = \mathbf{C}\mathbf{z}_2, \forall \mathbf{z}_2 \in \mathbb{R}^k\},$$

where $\mathbf{C} \in \mathbb{R}^{t\times k}$ is a matrix whose column form a basis of the nullspace of $\mathbf{J}^{\mathrm{T}}$ (whose nullity is k). The preload force subspace is thus given by

$$\mathcal{F}_{ho} = \mathcal{F}_h \cap \mathcal{F}_o = \mathcal{R}(\mathbf{A}) \cap \mathcal{R}(\mathbf{C}).$$

Proposition 4 *The set of passive internal forces $\mathcal{F}_{ho}$ is a linear subspace of $\mathbb{R}^t$, i.e., every passive internal force can be written as the product of a basis matrix $\mathbf{P}$ times an arbitrary coefficient vector $\mathbf{z}$ of suitable dimension.*

Proof. Since the desired set is the intersection of the range space of matrices $\mathbf{A}$ and $\mathbf{C}$, it is a linear subspace. To evaluate a basis, consider the equation $\mathbf{A}\mathbf{z}_1 = \mathbf{C}\mathbf{z}_2$, or, in matrix form,

$$[\mathbf{A}\ \ -\mathbf{C}]\begin{pmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{pmatrix} = 0. \tag{27}$$

Let $\mathbf{Q}_o = [\mathbf{A} \;\; -\mathbf{C}] \in \mathbb{R}^{t\times(h+k)}$, and let $\mathbf{B}_o \in \mathbb{R}^{(h+k)\times d}$ be a matrix whose columns span the nullspace of $\mathbf{Q}_o$ (whose nullity is d). Finally, partition $\mathbf{B}_o$ as $\mathbf{B}_o = \left[\mathbf{B}_{o1}^{\mathrm{T}} \;\; \mathbf{B}_{o2}^{\mathrm{T}}\right]^{\mathrm{T}}$, where $\mathbf{B}_{o1} \in \mathbb{R}^{h\times d}$, and $\mathbf{B}_{o2} \in \mathbb{R}^{k\times b}$. The desired subspace is thus obtained as

$$\mathcal{F}_{ho} = \mathcal{R}(\mathbf{A}\mathbf{B}_{o1}).$$

Therefore, all possible preload forces can be expressed as

$$\mathbf{t} = \mathbf{P}\mathbf{z}, \tag{28}$$

where the columns of the $t \times p$ matrix $\mathbf{P}$ form a basis of the range of $\mathbf{A}\mathbf{B}_{o1}$, and $\mathbf{z} \in \mathbb{R}^p$ parameterizes the preload subspace. □

Interestingly enough, the preload subspace description is not affected by the system stiffness matrix, $\mathbf{K}$. From such observation, it clearly follows that any attempt at deriving a basis of either the active or passive internal subspaces by using the concept of "orthogonal complement" to basis vectors of the other subspace, is misconceived. However, from the definition of the particular, active and preload homogeneous force subspaces follows

$$\mathcal{R}(\mathbf{P}) \oplus \mathcal{R}(\mathbf{E}) = \mathcal{N}(\mathbf{G}) \tag{29}$$

$$\mathcal{R}(\mathbf{P}) \oplus \mathcal{R}(\mathbf{E}) \oplus \mathcal{R}(\mathbf{G}_K^R) = \mathbb{R}^t. \tag{30}$$

6 Summary of Results

In view of the above discussion and results, a three–term description of contact force distribution in general manipulation systems can be given as

$$\mathbf{t} = -\mathbf{G}_K^R\mathbf{w} + \mathbf{E}\mathbf{y} + \mathbf{P}\mathbf{z}. \tag{31}$$

The first term in the right–hand side is a particular solution of the grasp balance equation. Using the right–inverse of the grasp matrix defined in (18), this term corresponds to the contact forces exerted on the object due to the external load $\mathbf{w}$, when internal forces are zero. This is an invariant, physically meaningful choice for the particular solution of the grasp balance equation. The second term in the right–hand side of (31) is a parameterized homogeneous solution corresponding to active internal forces. Optimal grasp force distributions can be found through minimizing a cost function with respect to $\mathbf{y}$. The third term in the right–hand side of (31) is a fixed homogeneous solution corresponding to contact forces that are preloaded at the beginning of the grasp operation (in most practical cases, it can be assumed $\mathbf{z} = 0$).

Consider for instance the problem of finding the optimal distribution of contact forces in the grasp of an object subject to external load $\mathbf{w}_o$, with regard to the minimization of power consumption in the actuators of the finger joints. A cost function can be written in terms of the joint torques and of a positive–definite weight matrix $\mathbf{W}$ as

$$V = \tau^{\mathrm{T}}\, \mathbf{W}\tau.$$

Note that $\tau = \mathbf{J}^{\mathrm{T}}\,(-\mathbf{G}_K^R\mathbf{w}_o + \mathbf{E}\mathbf{y} + \mathbf{P}\mathbf{z})$, and that the gradient of V with respect to $\mathbf{y}$ is

$$\frac{\partial V}{\partial \mathbf{y}} = \mathbf{E}^{\mathrm{T}}\, \mathbf{J}\mathbf{W}\mathbf{J}^{\mathrm{T}}\,(-\mathbf{G}_K^R\mathbf{w}_o + \mathbf{E}\mathbf{y} + \mathbf{P}\mathbf{z}),$$

If no constraints are in order for contact forces, the optimal grasp is obtained for

$$\hat{\mathbf{y}} = -(\mathbf{E}^{\mathrm{T}}\, \mathbf{J}\mathbf{W}\mathbf{J}^{\mathrm{T}}\, \mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}\, \mathbf{J}\mathbf{C}\mathbf{J}^{\mathrm{T}}\, \mathbf{G}_K^R\mathbf{w}_o,$$

In general, if unilateral and friction contact constraints are considered, this solution may result unfeasible. However, if the constraints are linearized in the form $\mathbf{C}\mathbf{t} \le \mathbf{b}$ (see e.g. Cheng and Orin [5]), the problem can be recast as a standard quadratic programming problem:

Find $\hat{\mathbf{y}}$ *such that*
$\gamma^{\mathrm{T}}\, \mathbf{y} + \mathbf{y}^{\mathrm{T}}\ ,\ \mathbf{y}$ *is minimum;*
$\hat{\mathbf{C}}\mathbf{y} \le \hat{\mathbf{b}}$,

where , $= \mathbf{E}^{\mathrm{T}}\, \mathbf{J}\mathbf{W}\mathbf{J}^{\mathrm{T}}\, \mathbf{E}$, $\gamma = 2(\mathbf{E}^{\mathrm{T}}\, \mathbf{J}\mathbf{W}\mathbf{J}^{\mathrm{T}}\,(\mathbf{P}\mathbf{z} - \mathbf{G}_K^R)$, $\hat{\mathbf{C}} = \mathbf{C}\mathbf{E}$, and $\hat{\mathbf{b}} = \mathbf{b} + \mathbf{C}(\mathbf{G}_K^R\mathbf{w} - \mathbf{P}\mathbf{z})$. As is well known, quadratic programming is a particularly tractable nonlinear programming problem, for which convergence to the solution in a finite number of iterations can be guaranteed. An efficient algorithm for this problem is, for instance, the Convex–Simplex–Method–Conjugate–Direction (CSM–CD) described by Zangwill [27]. Nakamura, Nagai, and Yoshikawa [19] solve an analogous problem without constraint linearization. A globally asymptotically convergent algorithm for the optimal choice of internal forces, with second–order rate of convergence, is proposed in [1].

From a practical viewpoint, calculation of $\mathbf{G}_K^R$ may be computationally too demanding to be incorporated in a real–time grasp control algorithm. Actually, any particular solution of (1) can be used as a starting point for an iterative optimization algorithm, and computationally efficient right–inverses (such as the $\{1\}$–inverse [14], will be in general preferable to both $\mathbf{G}_K^R$ and $\mathbf{G}^+$. The interest of (18) is more in the insight it gives, e.g. in choosing the optimal joint servo gains or in estimating poorly known structural compliances of the manipulation system.

7 Examples

In this section we will illustrate the above discussed algorithms and show how the manipulator kinematics and elasticity properties play an important role in the analysis of the grasp when general manipulation systems are considered. In order to do that, we will consider the grasp of the object depicted in fig.1–a by means of four different manipulation systems.

Let the contact points coordinates be $\mathbf{c}_1 = (0\ 0\ 2a)^{\mathrm{T}}$; $\mathbf{c}_2 = (0\ 2a\ 3a)^{\mathrm{T}}$; $\mathbf{c}_3 = (0\ 2a\ a)^{\mathrm{T}}$, and the corresponding unit normal vectors be $\mathbf{n}_1 = (0\ 1\ 0)^{\mathrm{T}}$; $\mathbf{n}_2 = (0\ -\frac{\sqrt{3}}{2}\ -\frac{1}{2})^{\mathrm{T}}$; and $\mathbf{n}_3 = (0\ -\frac{\sqrt{3}}{2}\ \frac{1}{2})^{\mathrm{T}}$. All contacts are modeled as "soft–finger". Accordingly, the dimension of composite contact force/torque vectors $\mathbf{t}$ is $t = 12$ and the grasp matrix results

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -2a & 0 & 0 & -3a & 2a & 0 & -a & 2a & 0 & 0 & 0 \\ 2a & 0 & 0 & 3a & 0 & 0 & a & 0 & 0 & 1 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ 0 & 0 & 0 & -2a & 0 & 0 & -2a & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

A basis of the null–space of $\mathbf{G}$ is provided by the columns of the matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & -2 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & -4a & \frac{2}{\sqrt{3}}a & 1 \\ 0 & 0 & 0 & 4a & \frac{2}{\sqrt{3}}a & 1 \end{pmatrix}.$$

Note that the first three columns correspond to contact forces taken two at a time and opposing each other along the edges of the grasp triangle. The presence of friction torques at the soft-finger contacts produces the last three columns of $\mathbf{A}$. The stiffness matrix $\mathbf{K}$ will be evaluated in each case according to (11). In our example, $\mathbf{C}_s$ is assumed diagonal, with $\mathbf{C}_{s_{j,j}} = 0.05$ mm/N for linear springs ($1 \le j \le 9$), and $\mathbf{C}_{s_{j,j}} = 0.01$ deg./Nmm for rotational springs ($10 \le j \le 12$). On the other hand, assuming that the q joints are controlled with q independent position servos with steady–state gain $k_p = 100.0$ Nmm/deg., we have $\mathbf{C}_q = \frac{1}{k_p}\mathbf{I}_q$.

7.1 Simple gripper

As a first case study, consider the simple one–joint gripper of fig.1–c. The joint axis is $\mathbf{z}_1 = (1\ 0\ 0)^{\mathrm{T}}$, and its origin $\mathbf{o}_1 = (0\ 0\ 0)^{\mathrm{T}}$. The jacobian matrix in this case is $\mathbf{J}^{\mathrm{T}} = (0\ -2a\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$. With the stiffness parameters above listed, the resulting stiffness matrix is

$$\mathbf{K} = \begin{pmatrix} 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{0.05+0.04a^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{pmatrix}$$

As is intuitively clear, the subspace of active internal forces is one–dimensional in this example, and the preload force subspace is 5–dimensional:

$$\mathbf{E} = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \qquad \mathbf{P} = \begin{pmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 1 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & -\sqrt{2} & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}a & 0 & \sqrt{3} \\ -4a & 0 & a & 0 & 1 \\ 4a & 0 & a & 0 & 1 \end{pmatrix}.$$

Recall that each column represents a combination of contact forces $\mathbf{t}_i$ and normal torques $m_i\mathbf{n}_i$ at the contact points, arranged as $(\mathbf{t}_1^{\mathrm{T}}\ \mathbf{t}_2^{\mathrm{T}}\ \mathbf{t}_3^{\mathrm{T}}\ m_1\ m_2\ m_3)^{\mathrm{T}}$. The only

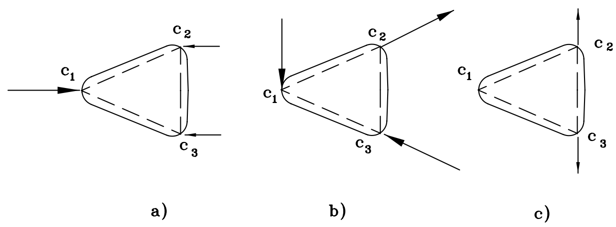


Figure 3: Active and preload internal forces for the grasp of fig.1

set of internal forces that can be modified at will is represented in fig.3–a. Fig.3–b and fig.3–c represent two of the basic combinations of passive internal forces (columns 2 and 4 of $\mathbf{P}$, respectively), that cannot be modified by joint control. To fix some ideas, let $a = 50$mm, and assume an external force of 1N in the y direction is applied at the point of contact points. The corresponding wrench is $\mathbf{w} = (0 \ 1\text{N} \ 0 \ -100\text{Nmm} \ 0 \ 0)^{\mathrm{T}}$. In the following, we will use coherently Newtons and millimeters as force and length units, and avoid explicit notation. The load distribution among contact forces is evaluated as $\mathbf{t}_p = -\mathbf{G}_K^R \mathbf{w} = (0 \text{ -0.0002 } 0 \ 0 \text{ -0.4999 } 0 \ 0 \text{ -0.4999 } 0 \ 0 \ 0 \ 0)^{\mathrm{T}}$. This result shows that the joint compliance greatly reduces the load share taken by the first contact. Also, note that the contact force at $\mathbf{c}_1$ is directed outside of the object. If adhesive contact forces are not allowed, as usually is the case, the pure particular solution yields an unstable grasp (non–compressive contact forces). However, by adding an appropriate active internal force, it is possible to avoid this problem. For instance, by choosing $\mathbf{y} = 0.01$, we have from (31) $\mathbf{t} = \mathbf{t}_p + \mathbf{E}\mathbf{y} = (0 \ 0.02 \ 0 \ 0 \text{ -0.51 } 0 \ 0 \text{ -0.51 } 0 \ 0 \ 0 \ 0)^{\mathrm{T}}$. To implement such correction, the position setpoints of the robot joint must be changed by $\Delta\mathbf{q} = -0.02$rad (see (26)). Note that this apparently large displacement is almost completely absorbed by servo compliance, and only minor changes in joint angle will actually occur (namely, $\Delta\mathbf{q} - \mathbf{C}_q \mathbf{J}^{\mathrm{T}} \mathbf{E}\mathbf{y} = -1.5 10^{-5}$rad). Correspondingly, the equilibrium position of the object is slightly displaced by $\Delta\mathbf{u} = (0 \ 0.5\, 10^{-3} \ 0 \ 0 \ 0 \ 0)^{\mathrm{T}}$.

7.2 Two–joint hand

Consider the two–joint hand of fig.4–a, which employs the two links and the palm to grasp the object of fig.1–a. Joint axes are $\mathbf{z}_1 = \mathbf{z}_2 = (1 \ 0 \ 0)^{\mathrm{T}}$, and the origins are $\mathbf{o}_1 = (0 \ 0 \ 0)^{\mathrm{T}}$,

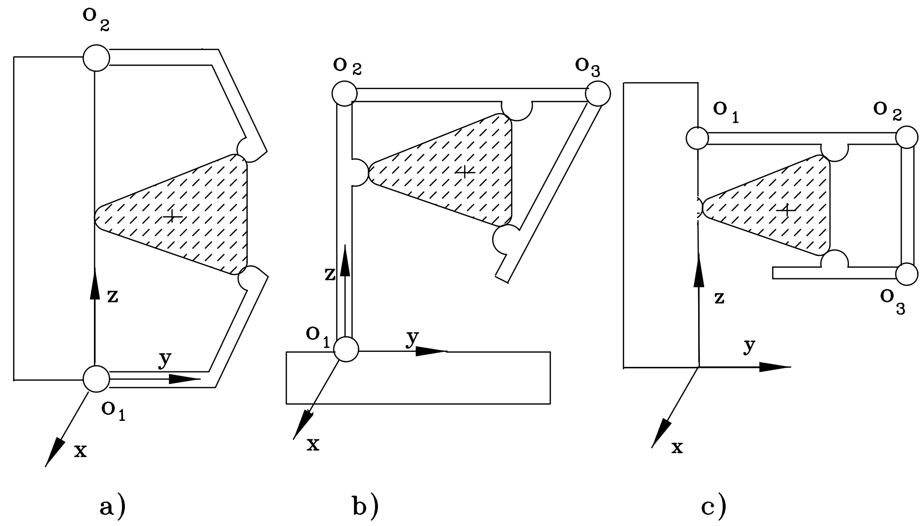


Figure 4: Three different manipulators grasping the same object.

and $\mathbf{o}_2 = (0\ 0\ 4a)^{\mathrm{T}}$. The jacobian and stiffness matrices are in this case (for $a = 50$mm):

$$\mathbf{J} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 50 \\ 0 & 100 \\ 0 & 0 \\ -50 & 0 \\ 100 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} ; \mathbf{K} = \begin{pmatrix} 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 16 & -8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -8 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{pmatrix}$$

The subspace of active, internal forces and preload contact forces are 2– and 4–dimensional, respectively, and their basis matrices are

$$\mathbf{E} = \begin{pmatrix} 0 & 0 \\ -2 & 0 \\ 0 & -2 \\ 0 & 0 \\ 1 & 2 \\ 6 & 1 \\ 0 & 0 \\ 1 & -2 \\ -6 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}; \qquad \mathbf{P} = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & \sqrt{2} & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -\sqrt{2} & 0 \\ 0 & -2 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & \sqrt{3} \\ -4 & 0 & 1 & 1 \\ 4 & 0 & 1 & 1 \end{pmatrix}.$$

The particular solution corresponding to an external load as in the case above is $\mathbf{t}_p = (0$ -0.38 0 0 -0.31 0.1 0 -0.31 -0.1 0 0 $0)^{\mathrm{T}}$. Again, the first contact force is adhesive, and must be corrected by setting, for instance, $\mathbf{y} = (-0.3\ 0)^{\mathrm{T}}$. From (31), $\mathbf{t} = \mathbf{t}_p + \mathbf{E}\mathbf{y} =$ (0 0.22 0 0 -0.61 -1.7 0 -0.61 1.7 0 0 $0)^{\mathrm{T}}$. The position setpoints of the robot joints must be changed by $\Delta\mathbf{q} = (1.95 - 1.95)$rad, and the equilibrium position of the object is displaced by $\Delta\mathbf{u} = (0$ -0.03 0 0 0 $0)^{\mathrm{T}}$.

7.3 Three–joint limb.

Consider now the three–joint limb depicted in fig.4–b, where $\mathbf{z}_1 = \mathbf{z}_2 = \mathbf{z}_3 = (1\ 0\ 0)^{\mathrm{T}}$, and $\mathbf{o}_1 = (0\ 0\ 0)^{\mathrm{T}}$, $\mathbf{o}_2 = (0\ 0\ 3a)^{\mathrm{T}}$, and $\mathbf{o}_3 = (0\ 3a\ 3a)^{\mathrm{T}}$. The jacobian and stiffness matrices are in this case (for $a = 50$mm):

$$\mathbf{J} = \begin{pmatrix} 0 & 0 & 0 \\ -100 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -150 & 0 & 0 \\ 100 & 100 & 0 \\ 0 & 0 & 0 \\ -50 & 100 & 100 \\ 100 & 100 & -50 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{K} = \begin{pmatrix} 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 14 & 0 & 0 & -8.8 & 1.9 & 0 & -0.63 & -1.3 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -8.8 & 0 & 0 & 6.8 & 2.8 & 0 & -0.94 & -1.9 & 0 & 0 & 0 \\ 0 & 1.9 & 0 & 0 & 2.8 & 12 & 0 & -4.1 & -8.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.63 & 0 & 0 & -0.94 & -4.1 & 0 & 1.4 & 2.7 & 0 & 0 & 0 \\ 0 & -1.3 & 0 & 0 & -1.9 & -8.2 & 0 & 2.7 & 5.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{pmatrix}$$

The dimension of the active and preload force subspaces are not changed in this case:

$$\mathbf{E} = \begin{pmatrix} 0 & 0 \\ -2 & 0 \\ 0 & 5 \\ 0 & 0 \\ 1 & -5 \\ 0 & 1 \\ 0 & 0 \\ 1 & 5 \\ 0 & -6 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}; \qquad \mathbf{P} = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & \sqrt{2} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 1 & 0 & -\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}a & \sqrt{3} \\ -4a & 0 & a & 1 \\ 4a & 0 & a & 1 \end{pmatrix}.$$

The particular solution corresponding to the external force of 1N applied at the point of contact points is $\mathbf{t}_p = (0\ \text{-}0.998\ 0.0006\ 0\ \text{-}0.0016\ \text{-}0.0001\ 0\ \text{-}0.0004\ \text{-}0.0007\ 0\ 0\ 0)$. To avoid breaking the first contact, a correction can be choosen as $\mathbf{y} = (-0.6\ 0)^{\mathrm{T}}$. Correspondingly, $\mathbf{t} = \mathbf{t}_p + \mathbf{E}\mathbf{y} = (0\ 0.20\ 0\ 0\ \text{-}0.6\ 0\ 0\ \text{-}0.6\ 0\ 0\ 0\ 0)^{\mathrm{T}}$. The position setpoints of the robot joints must be changed by $\Delta\mathbf{q} = (0\ -0.6\ -0.6)$rad, and the equilibrium position of the object is displaced by $\Delta\mathbf{u} = (0\ \text{-}0.24\ 0\ 0.001\ 0\ 0)^{\mathrm{T}}$.

7.4 Three–joint limb and chest.

If the object of fig.1–a is grasped by a three–joint limb and the chest of a robot such as depicted in fig.4–c, a three–dimensional subspace of realizable, internal forces can be obtained. In fact, assuming in this example $\mathbf{z}_1 = \mathbf{z}_2 = \mathbf{z}_3 = (1\ 0\ 0)^{\mathrm{T}}$, $\mathbf{o}_1 = (0\ 0\ 3a)^{\mathrm{T}}$,

$\mathbf{o}_2 = (0\ 3a\ 3a)^{\mathrm{T}}$, and $\mathbf{o}_3 = (0\ 3a\ a)^{\mathrm{T}}$, the jacobian and stiffness matrices result (for $a = 50\mathrm{mm}$):

$$\mathbf{J} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 100 & 0 & 0 \\ 0 & 0 & 0 \\ 100 & 100 & 0 \\ 100 & -50 & -50 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{K} = \begin{pmatrix} 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.11 & 0 & -0.04 & -0.06 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.04 & 0 & 0.02 & 0.02 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.06 & 0 & 0.02 & 0.04 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{pmatrix},$$

Correspondingly, the $\mathbf{E}$ and $\mathbf{P}$ basis matrices result

$$\mathbf{E} = \begin{pmatrix} 0 & 0 & 0 \\ -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \qquad \mathbf{P} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & \sqrt{2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}}a & \sqrt{3} \\ -4a & a & 1 \\ 4a & a & 1 \end{pmatrix}.$$

The particular solution corresponding to the above discussed external force is in this case $\mathbf{t}_p = (0\ \text{-}0.998\ 0.001\ 0\ \text{-}0.002\ \text{-}0.001\ 0\ 0\ 0\ 0\ 0\ 0)^{\mathrm{T}}$. Again, the first contact force is adhesive, and can be corrected by choosing e.g. $\mathbf{y} = (-3\ 2\ -3)^{\mathrm{T}}$. From (31), $\mathbf{t} = \mathbf{t}_p + \mathbf{E}\mathbf{y} = (0\ 5\ 2\ 0\ \text{-}5\ \text{-}3\ 0\ \text{-}1\ 1\ 0\ 0\ 0)^{\mathrm{T}}$. Accordingly, the position setpoints of the robot joints are changed by $\Delta\mathbf{q} = (-3\ -1.5\ -0.5)$rad, and the equilibrium position of the object is displaced by $\Delta\mathbf{u} = (0\ \text{-}1.4\ \text{-}0.1\ \text{-}0.01\ 0\ 0)^{\mathrm{T}}$.

7.5 Kerr and Roth's example

In an important early paper on grasp optimization, Kerr and Roth [11] discuss an example grasp by two fingers (fig. 5). They use linear programming techniques to choose the optimal combination of forces in the nullspace of the grasp matrix, in our notation:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \sqrt{2}/2 & 0 \\ 0 & 0 \\ 0 & 0 \\ \sqrt{2}/2 & 0 \\ 0 & \sqrt{2}/2 \\ 0 & \sqrt{2}/2 \end{bmatrix}.$$

However, the manipulation system is apparently defective, and only the span of the first column of $\mathbf{A}$ results actively modifiable through joint torque commands. Therefore, the optimization search should have been performed inside that span (corresponding to that of

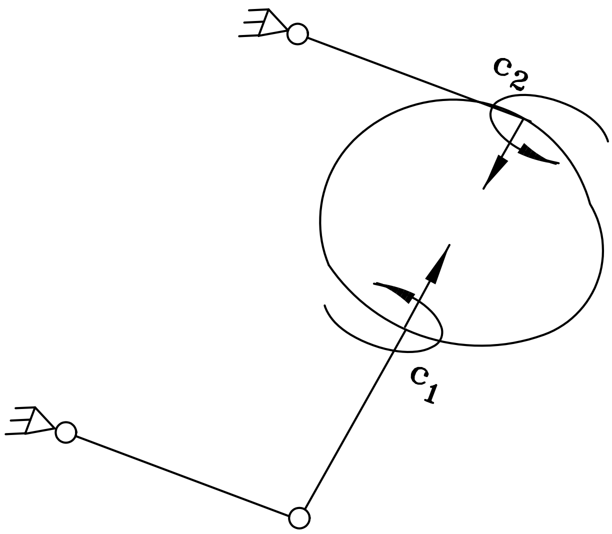


Figure 5: Kerr and Roth's [12] example no.1.

matrix $\mathbf{E}$ by the method above), to avoid possibly unfeasible results; moreover, reduction of the dimensionality of the search space is of great advantage for computational issues.

8 Conclusion

In this paper the problem of force decomposition in general manipulation systems, including multiple whole–limb cooperating manipulators, has been considered. An attempt is made at explaining the geometric structure of the vector space of contact forces and torques that are mutually exerted between the manipulation system links and the manipulated object. Three fundamental subspaces are described, corresponding to: contact forces that can be caused by external forces acting on the object, $\mathcal{R}(\mathbf{K}\mathbf{G}^{\mathrm{T}})$, see (18); internal contact forces directly realizable by joint commands, $\mathcal{R}(\mathbf{E})$; and internal contact forces that can only be set by preloading, $\mathcal{R}(\mathbf{P})$. Algorithms for the determination of a vector basis for such subspaces are also provided. These results, particularly those concerning the description of active internal forces, are necessary requisites to the realization of optimal grasp control algorithms with general manipulation systems.

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