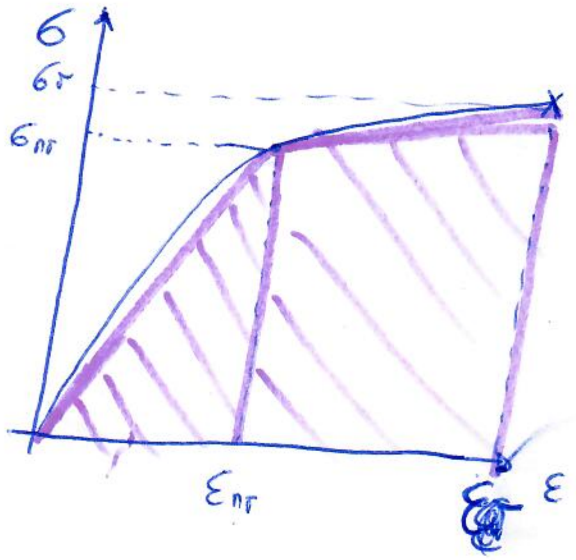
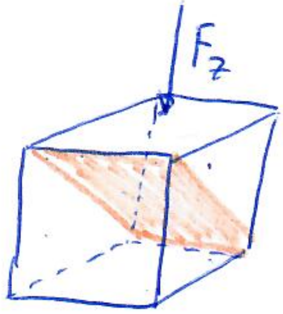


$$E = V_{oss}\left[\frac{1}{2}\,\varepsilon_{nr}\,\sigma_{nr} + (\varepsilon_r - \varepsilon_{nr})\cdot\sigma\right.$$

$$E^{El} = V_{osso}\cdot\frac{1}{2}\,\varepsilon_{nr}\,\sigma_{nr}$$

$$\sigma = \varepsilon\, E \qquad \underline{\sigma_{ij}} = \underline{E_{ijkl}}\,\underline{\varepsilon_{kl}}$$



$36 \cdot 6 = 216$

ortotropo $\quad$ xy, xz, zy $\quad 36 \cdot 3 = 108$

$t = t^*$ proprietà di ortotropia rest. costante nel temp.

$$\sigma_i^{6} = \underline{E_{ij}}\,\varepsilon_j^{6}$$

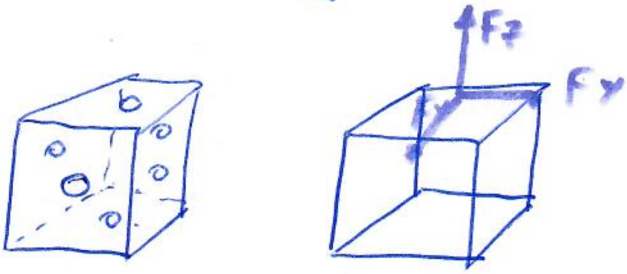
$$\begin{bmatrix} E_{11} & E_{12} & \cdots & E_{16} \\ E_{21} & E_{22} & \cdots & E_{26} \\ \vdots & \vdots & \ddots & \vdots \\ E_{61} & E_{62} & \cdots & E_{66} \end{bmatrix} \Rightarrow \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{16} \\ & E_{22} & \cdots & E_{26} \\ \emptyset & & \ddots & \vdots \\ & & & E_{66} \end{bmatrix}$$

36 $\qquad$ 21

$$\begin{bmatrix} E_{11} & E_{12} & \cdots & E_{16} \\ & E_{22} & \cdots & E_{26} \\ & \phi & \ddots & \vdots \\ & & & E_{66} \end{bmatrix} \Rightarrow \left[\begin{array}{c|c} A & B \\ \hline D & C \end{array}\right]$$

$$\left[\begin{array}{c|c} I_{DR} & \cancel{B}_{DR} - C \\ \hline C - I_{DR} & Coll \end{array}\right] = \left[\begin{array}{c|c} I_{DR} & \emptyset \\ \hline \emptyset & Coll \end{array}\right]$$

$$I_{DR} = \begin{bmatrix} I_{DR11} & I_{DR12} & I_{DR13} \\ 0 & I_{D22} & I_{DR23} \\ 0 & 0 & I_{DR33} \end{bmatrix}$$

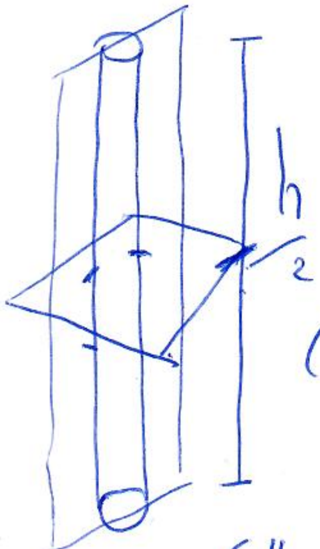


$$I_{DR\,12} = I_{DR\,23}$$

$$I_{DR} = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ 0 & E_{22} & E_{12} \\ 0 & 0 & E_{33} \end{bmatrix}$$

5 element

$$Coll = \begin{bmatrix} Coll_{11} & Coll_{12} & Coll_{13} \\ 0 & Coll_{22} & Coll_{23} \\ 0 & 0 & Coll_{33} \end{bmatrix}$$

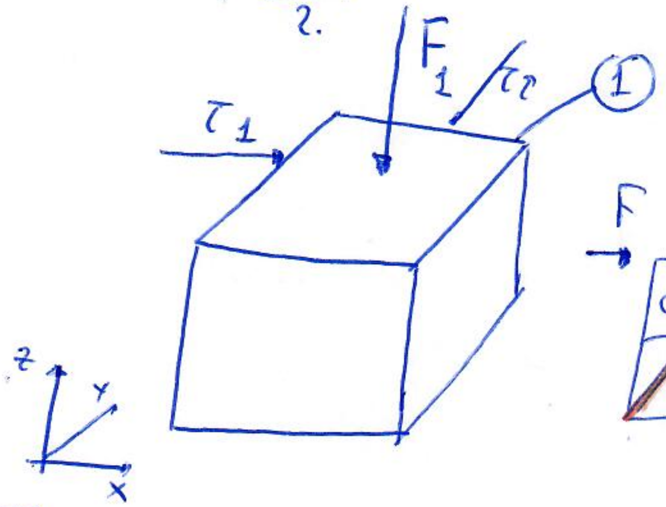


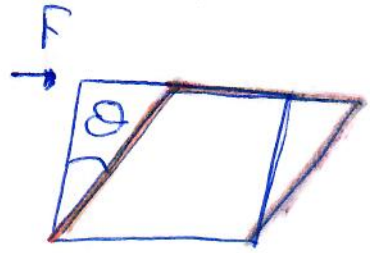
$$Coll = \begin{bmatrix} Coll_{11} & 0 & 0 \\ 0 & Coll_{22} & 0 \\ 0 & 0 & Coll_{33} \end{bmatrix}$$

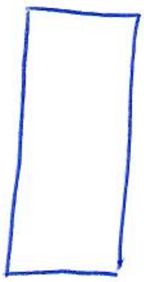
$$Coll_{33} = \frac{Coll_{11} + Coll_{22}}{2}$$

$$E = \begin{bmatrix} E_{11} & E_{12} & E_{13} & & 0 & \\ 0 & E_{22} & E_{12} & & & \\ 0 & 0 & E_{33} & & & \\ & & & E_{44} & 0 & 0 \\ & 0 & & 0 & E_{55} & 0 \\ & & & 0 & 0 & \frac{E_{44}+E_{55}}{2} \end{bmatrix}$$

7 element







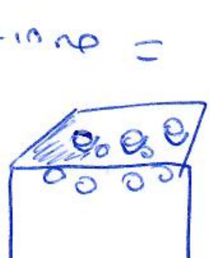
E_z
E_{xy}

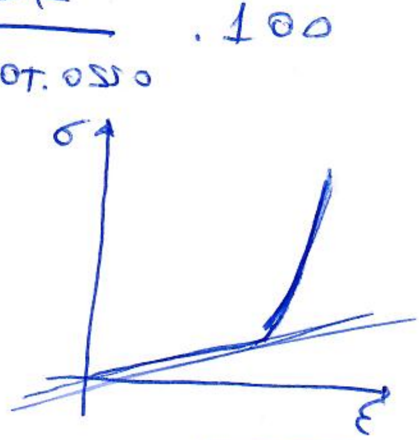
porosità

grado di mineralizzazione

$$\text{porosità} = \frac{\text{Vol. vuoto}}{\text{Vol. totale osso}} = p$$

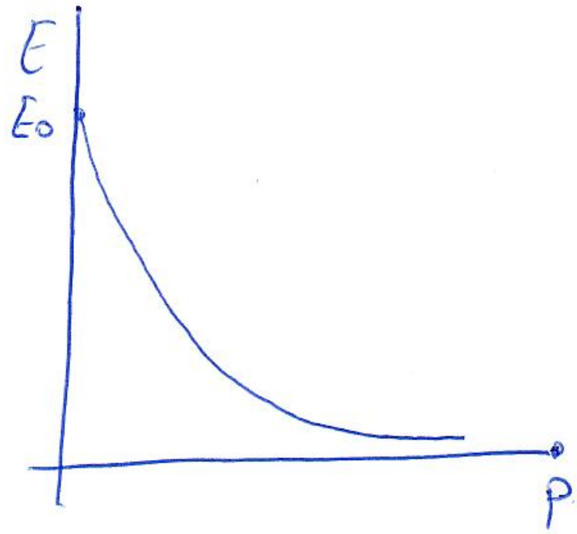
$$\text{grado di mineralizzazione} = \frac{V_{IDR}}{V_{Tot.\,osso}} \cdot 100$$





$$E = E_0 (1-p)^{\alpha}$$

$5 < \alpha < 10$ $\quad \boxed{\alpha = 5}$

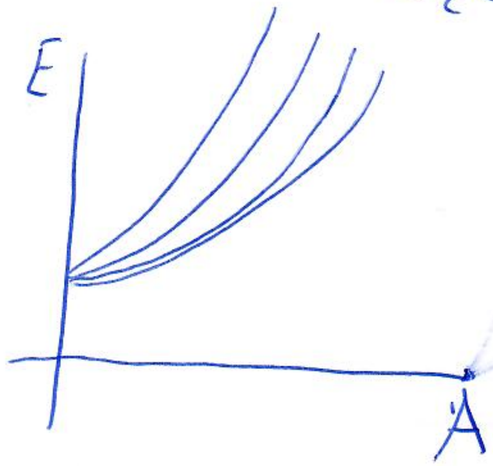


$$E_1 = E_0 (1-p)^{\alpha} \quad \alpha = 5$$

$$E_2 = E_1 A^{\beta} \quad 1 < \beta < 5$$

$$\beta = 1$$

$$E_2 = E_0 (1-p)^{\alpha} A^{\beta}$$



$$V_{osso} = V_M + V_V$$

$$\rho = \frac{M}{V_{osso}}$$

$$M = \rho V_{osso} = \rho [V_M + V_V]$$

$$M = \rho \left[\frac{V_M}{V_{osso}} \cdot V_{osso} + \frac{V_V}{V_{osso}} \cdot V_{osso} \right] = \rho \left[f_{MAT} + \underset{\underset{p}{\parallel}}{f_{viva}} \right] V_{osso}$$

$$M = \rho V_{osso} [f_{MAT} + p]$$

$$\rho = \frac{M_{osso}}{V_{osso}} = \frac{M_{MAT} + M_{viv}}{V_{osso}} = \frac{M_{MAT}}{V_{MAT}} \cdot \frac{V_{MAT}}{V_{osso}} + \frac{M_{viv}}{V_{viv}} \cdot \frac{V_{viv}}{V_{osso}}$$

$$= \rho_{MAT} f_p + \rho_{viv} \cdot p$$

ρ_{MAT} = densità apparente osso

$$\rho_{MAT} = \frac{m_{MAT}}{V_{MAT}} = \frac{m_{idr} + m_{coll}}{V_{MAT}} = \frac{m_{idr}}{V_{idr}} \cdot \frac{V_{idr}}{V_{MAT}} + \frac{m_{coll}}{V_{col}} \cdot \frac{V_{coll}}{V_{MAT}}$$

$$= \rho_{idr} \, f_{idr} + \rho_{coll} \cdot f_{coll}$$

$\rho_{idr} \downarrow$ ash dry density

$$E_3 = E_2 \, \delta_{app}^{\gamma} \, \dot{\varepsilon} = E_1 \, A^{\beta} \, \delta_{app}^{\gamma} \, \dot{\varepsilon} = E_0 (1-p)^{\alpha} A^{\beta} \delta_{app}^{\gamma} \dot{\varepsilon}$$

$5 < \alpha < 10$ $\quad$ $\alpha = 5$

$1 < \beta < 5$ $\quad$ $\beta = 1$

$0.1 < \gamma < 1$ $\quad$ $\gamma = 1$

$p \longrightarrow$ TAC

$A \longrightarrow$ Risonanza magnetica funzionale

δ fosfatasi alcalina

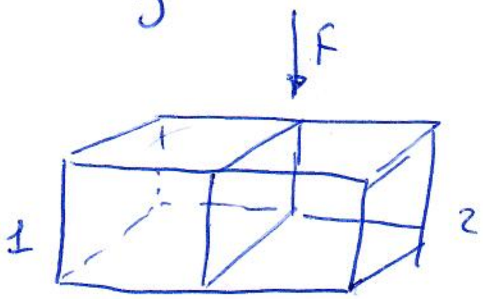
$E_{osso} > 1$ GPa $\quad$ protesi PRESS-fit

$0.5 < E_{osso} \leq 1$ GPa $\quad$ protesi cementata

$0.1 \leq E_{osso} \leq 0.5$ GPa $\quad$ osso cementificato

$E_{osso} < 0.1$ GPa $\quad$ non può essere dimensionata

Voigt



E_1

E_2

$A = A_1 + A_2$

$F = F_1 + F_2$

$$\frac{F}{A} = \frac{F_1}{A} + \frac{F_2}{A} = \frac{F_1}{A}\cdot\frac{A_1}{A_1} + \frac{F_2 \cdot A_2}{A \cdot A_2} = \frac{F_1}{A_1}\cdot f_1 + \frac{F_2}{A_2}\cdot f_2$$

$V_1 = A_1 \cdot h \quad V_2 = A_2 \cdot h \quad V_{TOT} = A \cdot h$

$$f_1 = \frac{V_1}{V_{TOT}} = \frac{A_1 \cdot h}{A \cdot h} = \frac{A_1}{A}$$

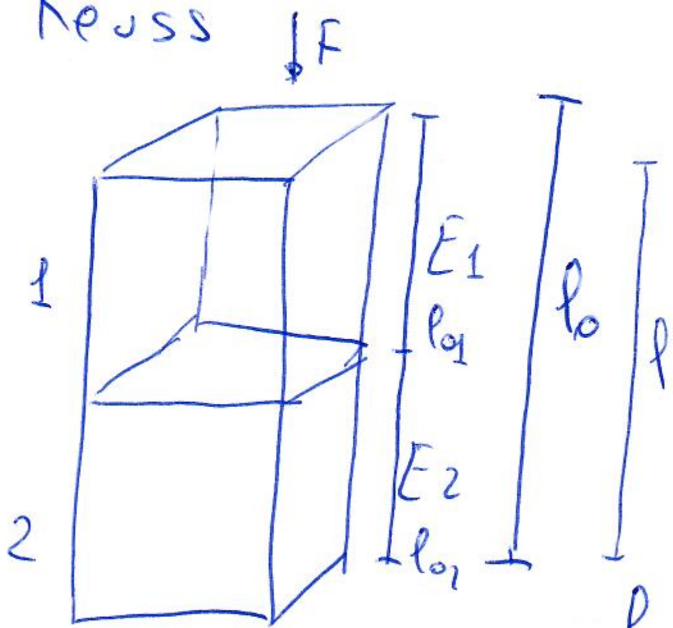
$$f_2 = \frac{V_2}{V_{TOT}} = \frac{A_2 \cdot h}{A \cdot h} = \frac{A_2}{A}$$

$$\sigma = \sigma_1 \cdot f_1 + \sigma_2 \cdot f_2$$

$$\varepsilon E_T = \varepsilon_1 E_1 f_1 + \varepsilon_2 E_2 f_2 \qquad \varepsilon = \varepsilon_1 = \varepsilon_2$$

$$\underline{E_T = E_1 f_1 + E_2 f_2} \quad \text{Voigt}$$

Reuss



$$l_0 = l_{01} + l_{02}$$

$$l = l_1 + l_2$$

$$l - l_0 = (l_1 - l_{01}) + (l_2 - l_{02})$$

$$\frac{l - l_0}{l_0} = \frac{l_1 - l_{01}}{l_0} + \frac{l_2 - l_{02}}{l_0} = \frac{l_1 - l_{01}}{l_0} \cdot \frac{l_{01}}{l_{01}} + \frac{l_2 - l_{02}}{l_0} \cdot \frac{l_{02}}{l_{02}}$$

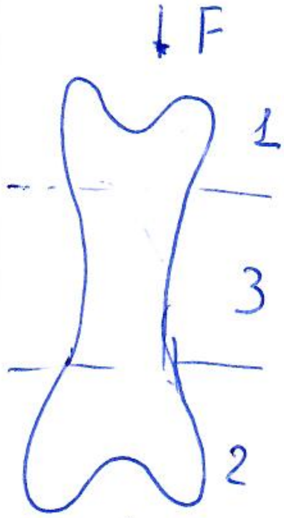
$$f_1 = \frac{V_1}{V_{TOT}} = \frac{A \cdot l_{01}}{A \cdot l_0} = \frac{l_{01}}{l_0} \qquad f_2 = \frac{V_2}{V_{TOT}} = \frac{A \cdot l_{02}}{A \cdot l_0} = \frac{l_{02}}{l_0}$$

$$\frac{l - l_0}{l_0} = \frac{l_1 - l_{01}}{l_{01}} f_1 + \frac{l_2 - l_{02}}{l_{02}} f_2$$

$$\varepsilon_{TOT} = \varepsilon_1 f_1 + \varepsilon_2 f_2 \qquad \sigma = \varepsilon E \qquad \varepsilon = \frac{\sigma}{E}$$

$$\frac{\sigma}{E_{TOT}} = \frac{\sigma_1}{E_1} \cdot f_1 + \frac{\sigma_2 f_2}{E_2} \qquad \sigma = \sigma_1 = \sigma_2$$

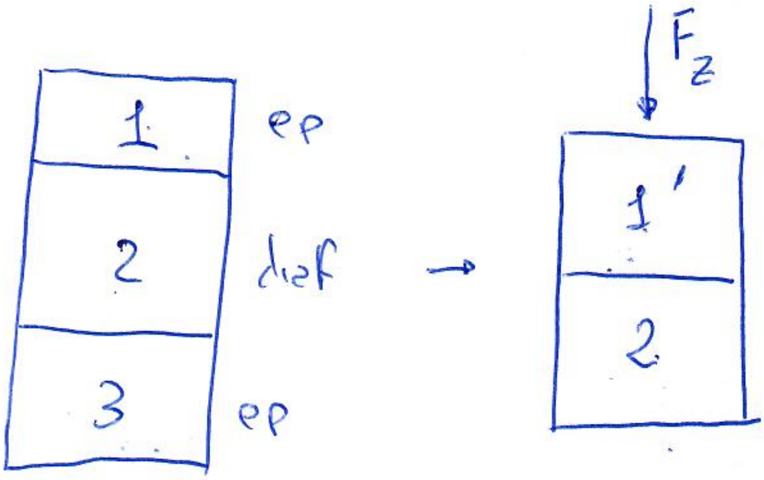
$$\frac{1}{E_{TOT}} = \frac{f_1}{E_1} + \frac{f_2}{E_2} \Rightarrow \boxed{E_{TOT} = \frac{E_1 \cdot E_2}{f_1 E_2 + f_2 E_1}}$$



$E_{osp} = 500\ MPa = 0.5\ GPa$

$E_z^{comp} = 17\ GPa$

$E_{xy}^{comp} = 12\ GPa.$

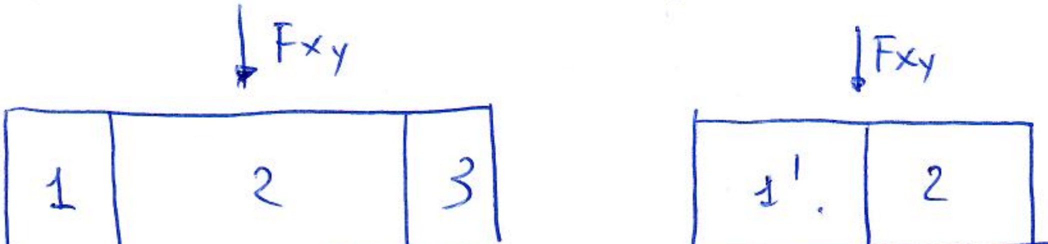


$1' = 1 + 3$

$$E_{TOT} = \frac{E_1' \cdot E_2}{f_1' E_2 + f_2 E_1'}$$

$f_{osp}^{ep} = 15\%$ $\quad f_{osp} = 30\%$ $\quad f_{diaf} = 70\%$

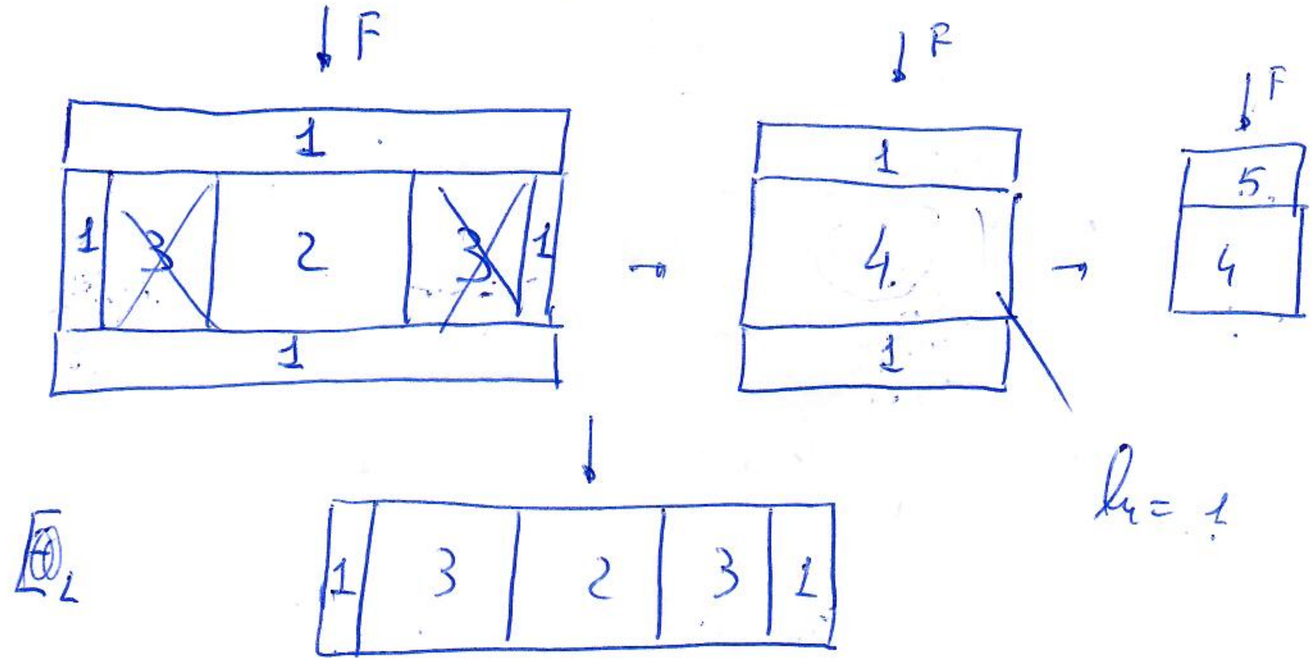
$$E_{TOT} = \frac{0.5 \cdot 17}{0.3 \cdot 17 + 0.7 \cdot 0.5} = \frac{8.5}{5.1 + 0.35} = \frac{8.5}{5.45} \simeq 1.6\ GPa$$



$$E_T = f_1' E_1' + f_2 \cdot E_2 = 0.3 \cdot 0.5 + 0.7\ 12 = 0.15 + 8.4 =$$
$$= 8.55\ GPa.$$

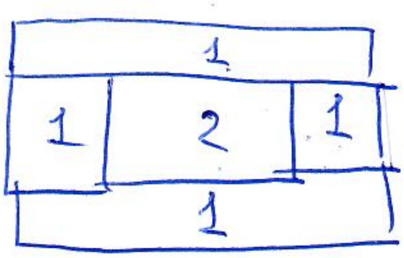
$\underline{f_{os} + f_{comp} = 1}$

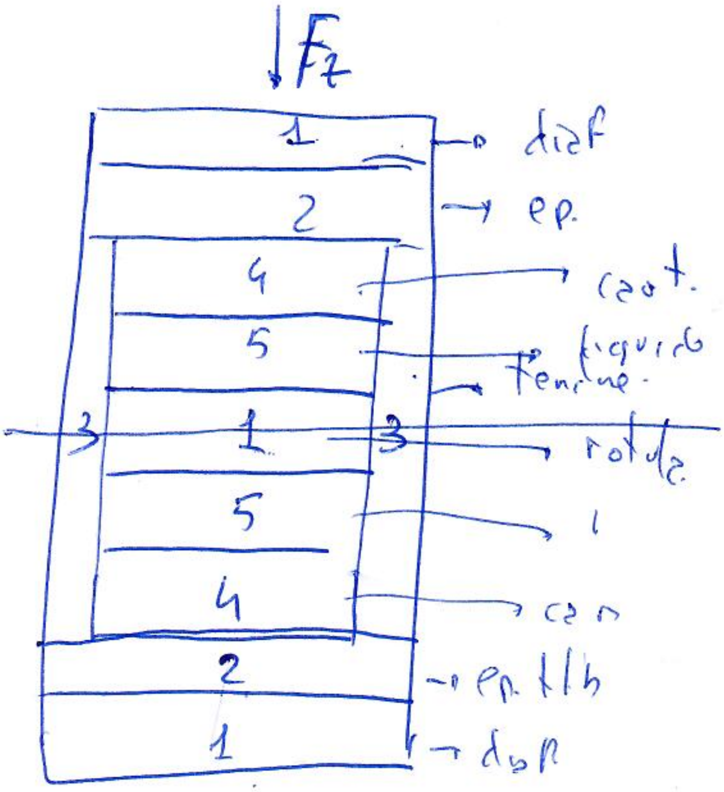
osso	f comp	f oss
femore	70%	30%
spalla	80%	20%
ulna, radio	90%	10%
tibia	80%	20%
mandibola	2%	98%

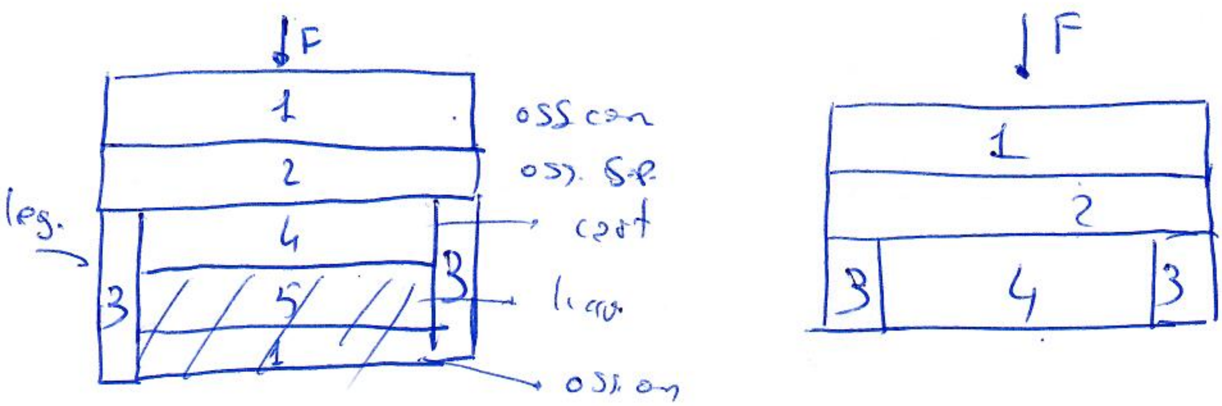


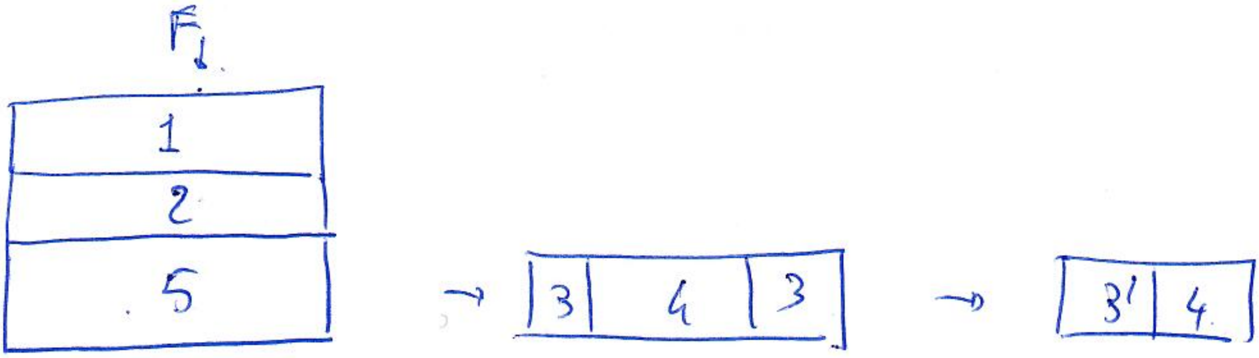
$$E_4 = 2 l_1 E_1 + 2 l_3 E_3 + l_2 E_2$$

$$\underline{E_{TOT}} = \frac{E_4 \cdot E_5}{l_5 E_4 + l_4 \cdot E_5} = \frac{(2 l_1 E_1 + 2 l_3 E_3 + l_2 E_2) \cdot E_1}{2 l_1 \cdot (2 l_1 E_1 + 2 l_3 E_3 + l_2 E_2) + (2 l_1 + 2 l_3 + l_2) \cdot E_1}$$







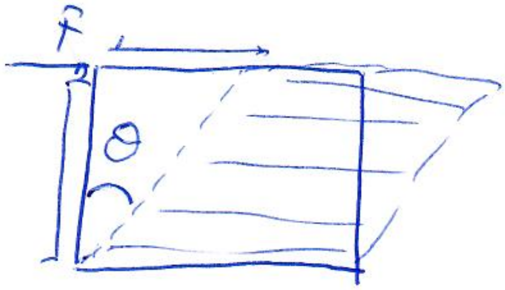


$$\frac{1}{E_{TOT}} = \frac{f_1}{E_1} + \frac{f_2}{E_2} + \frac{f_5}{E_5} = \frac{f_1}{E_1} + \frac{f_2}{E_2} + \frac{f_5}{f_3' E_3 + f_4 E_4}$$

$$f_1 + f_2 + f_5 = 1$$

$$f_1 + f_2 + 2f_3 + f_4 = 1$$

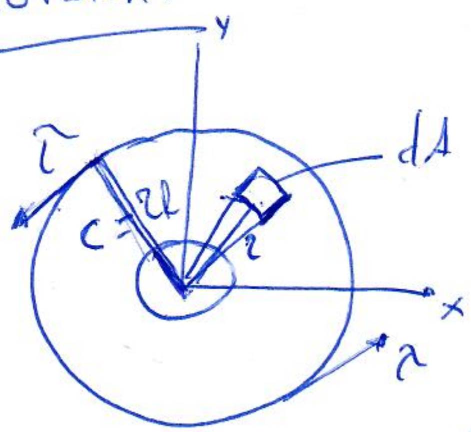
taglio



$$\varepsilon_{taglio} = \frac{x}{y} = \underline{tg\,\theta}$$

$$\underline{\sigma_{taglio}} = \underline{\varepsilon_{taglio}}\; E_t$$

Torsione



$$\underline{\sigma_{Torsione}} = \frac{\tau \cdot c}{I_{Tot}}$$

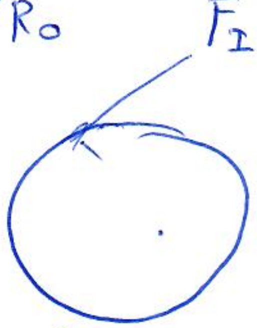
$$I_{TOT} = \sum r^2 dA = \iint r^2 dA$$

$$dA = dr\, r \cdot d\theta$$

$$I_{TOT} = \iint r^2 \cdot r\, dr\, d\theta =$$

$$I_{TOT} = \int_{R_0}^{R_1} r^3\, dr \cdot \int_0^{2\pi} d\theta = 2\pi \int_{R_0}^{R_1} r^3\, dr = 2\pi\, \frac{R_1^4}{4} - \frac{R_0^4}{4}$$

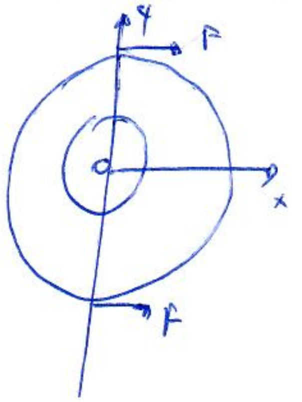
$$I_{TOT} = \frac{\pi}{2}\left(R_1^4 - R_0^4\right)$$



$$\tau = F_I \cdot c$$

$$\sigma_{TORS} = \frac{F_I \cdot c^2}{I_{TOT}}$$

bending → Flesson



$$\sigma_{ben} = \frac{M_{ben} \cdot y}{J_{cros}}$$

$$J = \sum y^2 dA = \iint y^2 dA = \iint y^2 r\,dr\,d\theta$$

$$y = r\cos\theta$$

$$J = \iint r^2 \cos^2\theta \cdot r\,dr\,d\theta = \int_{R_0}^{R_1} r^3\,dr \int_0^{2\pi} \cos^2\theta\,d\theta$$

$$J = \frac{R^4}{4}\Big|_{R_0}^{R_1} \int_0^{2\pi} \cos^2\theta\,d\theta = \frac{\pi}{4}\left(R_1^4 - R_0^4\right)$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\int_0^{2\pi} \frac{1+\cos 2\theta}{2}\,d\theta = \underbrace{\int_0^{2\pi} \frac{1}{2}\,d\theta}_{=\pi} + \underbrace{\int_0^{2\pi} \frac{\cos 2\theta}{2}\,d\theta}_{=\emptyset}$$

$$\sigma_{ben} = \frac{F \cdot y^2}{\frac{\pi}{4}\left(R_1^4 - R_0^4\right)}$$