Sample-Based Motion Planning for Robot Manipulators with Closed Kinematic Chains

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***Abstract*— Random sampling-based methods for motion planning of constrained robot manipulators have been widely studied in recent years. The main problem to deal with is the lack of an explicit parametrization of the non linear submanifold in the Configuration Space ($\mathcal{CS}$) imposed by the constraints in the system. Most of the proposed planning methods use projections to generate valid configurations of the system slowing the planning process.**

Recently, new robot mechanism includes compliance either in the structure or in the controllers. In this kind of robot most of the times the planned trajectories are not executed exactly due to uncertainties and interactions with the environment. Indeed, controller references are generated such that the constraint is violated to indirectly generate forces during interactions.

With the purpose of avoiding projections, in this paper we take advantage of the compliance of systems to *relax* the geometric constraints imposed by closed kinematic chains. The relaxed constraint is then used in a state-of-the-art sub-optimal random sampling based technique to generate paths for constrained robot manipulators. As a consequence of relaxation, arising contact forces acting on the constraint change from configuration to configuration during the planned path. Those forces can be regulated using a proper controller that takes advantage of the geometric decoupling of the subspaces describing constrained rigid-body motions of the mechanism and the controllable forces.

I. INTRODUCTION

In robot motion planning, interacting with the environment is normally considered a task to avoid, however in everyday tasks humans don't do that. Actually, simple tasks as opening a door, sliding an object on a table and moving an object consist on taking advantage of the objects and their constraints with the environment rather than avoiding touching them. In robotics solving the problem of generating motions is not simple mainly because we need to face two main problems: 1) working on high dimensional spaces, which make the problem difficult to solve it optimally, and 2) working under constraints such as closed loop kinematic chains and force/torque limits. The first, is solved in an efficient way by randomly sampling the configuration space ($\mathcal{CS}$) of the robot. This is possible thanks to the available explicit description of the $\mathcal{CS}$. The second problem is harder due the fact that an explicit description of the admissible $\mathcal{CS}$ is not available. It means that not all random samples of the $\mathcal{CS}$ can be considered as a possible configuration to explore. There exist some approaches to generate motions for robots under environmental constraints, particularly closed kinematic chains, which are based either on the decomposition of the chain in a passive and an active part [1], or in the projection of any random sample to the admissible $\mathcal{CS}$ [2], [3].

In this paper we propose a new method to generate motions for robots manipulators under closed . It is based on the relaxation of the constraint to be able to randomly sample an augmented admissible $\mathcal{CS}$. Then, using state-of-the-art algorithms as RRT* [4], we guarantee the convergence of the algorithm to a path, connecting two points, which optimizes the distance to the constraint at each point on it.

Fig. 1. The Motion planning method presented in this paper is developed for systems with compliance either in their structure of via the control loop. In this case the bimanual system includes the compliance in the qbmotors [5] composing the structure which is combined with mechanically embedded compliance of the PISA/IIT SofHands [6].

A. Planning with Closed Kinematic Chains - State of the Art

Since the introduction of random sampling techniques for path planning, a lot of advances have been made in this field. There exist two main approaches in this topic, the first is the Probabilistic Roadmap (PRM) and the second is the Rapidly-Exploring Random Tree (RRT) introduced in [7] and [8] respectively. These two approaches were designed to plan motions in high dimensional spaces, in fact they are normally applied in the $\mathcal{CS}$ of robot manipulators. The major advances have been focused in the improvement of these methods to mainly include heuristics to speed up the planning time and bias the solutions to get preferred behaviors. For example in [9] exploration and exploitation of $\mathcal{CS}$ is balanced for fast convergence of the planners. In [10] the authors propose to include different heuristics to bias the growth of the trees towards a preferred part in the $\mathcal{CS}$.

The last major contribution in probabilistic motion planing

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was presented in [4] where the authors studied the quality of the paths generated by randomized planners. They proposed a modification of the RRT and PRM algorithms, called RRT* and PRM*, to generate better quality paths. The completeness and sub-optimality of the solutions are guaranteed. Some improvements to speed up the solutions of this planner have been proposed in [11] and [12].

The inclusion of constraints in random-sample based planners is another research line in the area, for example 1) *nonholonomic constraints* for mobile robots as summarized in [13], 2) *task constraints* where the end effector has to maintain a desired orientation over the whole planned path (for example a robot holding an object upright), [14], and 3) *closed kinematic chains* for cooperative robots or parallel manipulators [15]. The latter is sometimes considered a particular case of 2). There is also a research line to include *dynamic constraints* such as joint torque limits. This planning techniques are called kinodynamic motion planning [16]. In this work we will focus the attention to motion planing for systems with tasks space constraints, particularly closed kinematic chains generated by multiple robot object manipulation.

The main problem in motion planning for closed kinematic chains is that the admissible $\mathcal{CS}$ of the robot is nonlinear submanifold $\mathcal{M}_{\text{v}}$, described by the constraint equations, living in $\mathcal{CS}$. Particularly, all randomized planners include a function called *Sample* where a random point in the configuration space is generated. In case of closed kinematic chains the function *Sample* must return a random point on the aforementioned submanifold. The probability to do this is 0 because the manifold is a zero measure set in the configuration space.

B. *Path Execution*

Another problem to deal with in multiple robot manipulation is that during path execution suitable interaction forces must be ensured. This can be addressed with a suitable force/position controller using the theory presented in [17]. In that paper, the authors demonstrate that the object trajectories and the contact forces can be addressed as decoupled control problems. It implies that we can execute any object trajectory coming from planning phase while contact forces are steered so as to avoid violation of contact constraints, allowing to regulate a desired force during motion.

Inspired by the combination of the state of the art robots, such as the one in Fig. 1 which have a compliant rather than a rigid structure, and from the fact that force and position subspaces can be geometrically decoupled. In this paper we propose a new approach to generate motions for closed kinematic chains by transforming the lower dimensional submanifold into a narrow but fully dimensional volume so that the probability of sampling a point randomly on it is not null.

Due to computational considerations, in Jacobian projection based methods, a threshold to decide whether a new configuration is already in the manifold is defined by the user.

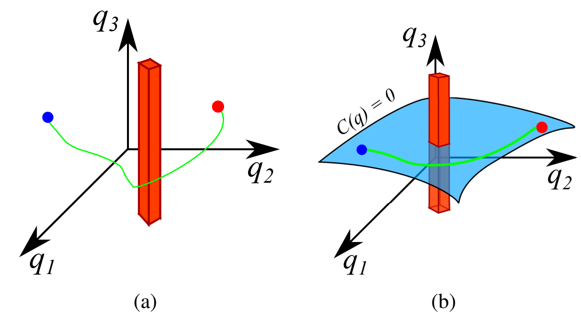


Fig. 2. Differences of motion planning problem without constraints (a) and with constraints (b). Initial position q_{init} in blue, final position q_{final} in red and Planned path in green. Constraint $C(q)=0$ in baby blue.

This threshold can be considered also as a relaxation which in this paper is formally addressed as interaction forces.

II. Organization

Section III formally defines the motion planning problem under task constraints and presents the main contribution of this work. In section IV the algorithm called soft-RRT* is presented, it describes the strategy implemented to find paths in the relaxed constraint. Section V addresses the problem of the practical implementations of the soft-RRT* algorithm and introduces a solution. After an example presented in section VI, section VII exposes the conclusions and future developments of this work.

III. Problem Definition

In this Section we formally introduce the motion planning problem for systems subject to constraints.

A. *Motion Planning Problem of Systems Under Constraints*

Consider a configuration space $\mathcal{M} \in \mathbb{R}^d$ that is a compact set of configurations q. Let $\mathcal{O} \in \mathcal{M}$ be the obstacle region and $\mathcal{M}_{\text{free}} := \mathcal{M} \setminus \mathcal{O}$ the configuration set free of obstacles. Introducing a kinematic constraint $C(q) = 0$ that limits the robot configurations and hence motion, see Fig. 2(b), we define a nonlinear submanifold in $\mathcal{M}$ as $\mathcal{M}_{\text{v}} := \{q : q \in \mathcal{M}_{\text{free}}, C(q) = 0\}$ to describe all configuration where none of the links of the mechanism collide neither with objects in the environment not with other links and satisfy the constraint. The motion planning problem is to find a continuous path $\sigma : [0,1] \to \mathcal{M}_{\text{v}}$ with $\{\sigma(0) = q_{\text{init}},\ \sigma(1) = q_{\text{final}}\}$.

As mentioned, the main challenge in applying sampling based motion planning algorithms to closed kinematic chains is that the probability of getting a random point laying on the submanifold $\mathcal{M}_{\text{v}}$ is zero.

B. *Relaxing Constraints*

In this paper we consider systems with compliance, it is introduced in the planning phase as a parameter to relax the constraint and to obtain $C(q) \leq \epsilon$. In the case in which the constraint is violated a proportional force f_h arises between the two parts in contact. With the inclusion of the parameter ϵ the submanifold describing the relaxed constraint can be considered as a space with the same dimension of $\mathcal{CS}$.

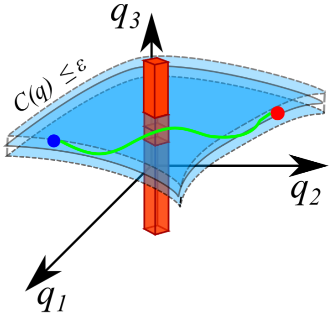
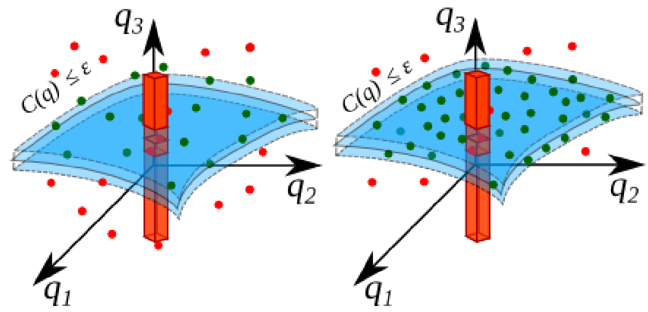


Fig. 3. Motion planning problem under relaxed constraints. Initial position q_{init} in blue. Final position q_{final} in red. Planned path in green. Constraint $C(q)$ in baby blue.



(a) (b)

Fig. 4. Graphical explanation of the difference of using a) uniform distribution to sample $\mathcal{M}$ and b) applying the algorithm presented in [18] to bias new samples to $\mathcal{M}_{\text{v}}$.

Thanks to this we can use rejection techniques to randomly sample the $\mathcal{CS}$ valid, now defined as $\mathcal{M}_{\text{r}} := \{q : q \in \mathcal{M}_{\text{free}}, C(q) \leq \epsilon\}$, and thus speed up the planning process. Now the planning problem is to find a continuous path $\sigma : [0,1] \to \mathcal{M}_{\text{r}}$, with, $\{\sigma(0) = q_{\text{init}},\ \sigma(1) = q_{\text{final}}\}$. This relaxed problem is graphically described in Fig. 3.

IV. Randomized Planning Algorithm

The random based-sampling algorithm used in this paper is the soft-RRT* reported in the algorithm 1. The difference with the original RRT* algorithm is that instead of just checking for collision we also check if the new configuration is inside the relaxed constraint.

Algorithm 1 $\mathcal{T} = (V, E) \leftarrow$ soft-RRT*(x_{init})

$\mathcal{T} \leftarrow InitTree()$;
for $i = 1$ *to* N **do**
  $x_{\text{rand}} \leftarrow$ **Sample**(i);
  $x_{\textbf{nearest}} \leftarrow$ **Nearest**(V,$x_{\textbf{rand}}$)
  $x_{\textbf{new}} \leftarrow$ **Steer**($x_{\textbf{nearest}}$, $x_{\textbf{rand}}$)
  if Constraints($x_{\textbf{nearest}}$, $x_{\textbf{new}}$) **then**
    $X_{\textbf{near}} \leftarrow$ **Near**($\mathcal{T}$, $x_{\textbf{new}}$)
    $x_{\textbf{min}} \leftarrow$**BestParent**($X_{\textbf{near}}$, $x_{\textbf{new}}$)
    $\mathcal{T} \leftarrow \mathcal{T} \cup$ ($x_{\textbf{new}}$,$x_{\textbf{min}}$)
    $\mathcal{T} \leftarrow$ **Rewire**($\mathcal{T}$, $X_{\textbf{near}}$, $x_{\textbf{new}}$)
  end if
end for
return $G = (V, E)$.

The main functions use in the Algorithm 1 are

- in function **Sample**, a configuration x_{rand} is generated using the algorithm presented in section IV-A, which converges to a uniform distribution of random points within the boundary layer.
- function **Nearest** returns the (previously sampled) configuration x_{nearest} closest to x_{rand}.
- function **Steer** connects two configurations if possible otherwise a configuration x_{new} is obtained as in RRT*. In this work we are using simple interpolation in joint space as a local steering procedure.
- **Near** function returns a set X_{near} containing points which are inside a ball centered on x_{near}. For details on the parameterization of the ball the reader can refer to [4].
- The main difference with the original RRT* algorithm is in the **Constraints** function. In our case this function includes not only collision checking but also the validation of the configuration x_{new} to be in the boundary layer. In this function the constrained optimization problem described in the subsection IV-B is solved
- **BestParent** function select the best configuration $x_{\text{min}} \in X_{\text{near}}$ to connect with x_{new}.
- Function **Rewire** rearrange the tree if one of the configurations on X_{near} could be better connected to the tree passing through x_{new}.

The cost function to optimize in the soft-RRT* is described by the euclidean distance in the $\mathcal{CS}$.

A. Biased Random Sampling

The first step in randomized path planners is performed in the function **Sample** and it consists on generating a new sample in $\mathcal{M}$. Typically, random configurations are taken using a uniform distribution to explore equally all regions in $\mathcal{CS}$. Doing the same in our problem, the probability of getting a new point in $\mathcal{M}_{\text{r}}$ can be computed as

$$\rho = \text{volume}(\mathcal{M}_{\text{r}})/\text{volume}(\mathcal{M}), \tag{1}$$

where the volume of $\mathcal{M}$ is defined by the mechanism, more precisely by the range of motion of all joints. On the other hand, the volume $\mathcal{M}_{\text{r}}$ is proportional to the relaxing parameter ϵ. As a consequence, the probability of getting a new point goes to 0 as ϵ approaches 0, in other words it means that bigger is ϵ, higher the probability of getting a new configuration in $\mathcal{M}_{\text{r}}$.

In practice, the parameter ϵ is associated with internal forces $f_{\text{h}} = K\epsilon$ where K can be interpreted as a suitable stiffness matrix resulted from contact and joint stiffness. Hence, the parameter ϵ depends on the compliance in the system and is limited by the user defined bounds for the forces f_{h}.

It is evident that if ϵ is small most of the new samples will be rejected in the **Constraints** function because they are not in the relaxed constraint. To minimize the impact of this fact, in the soft-RRT* we used the algorithm presented in [18] which builds an adaptive kd-tree to bias the random sampling procedure to converge to a uniform distribution not in $\mathcal{M}$ but in $\mathcal{M}_\mathrm{r}$. This algorithm works building a data structure to collect information about whether previously generated samples were or not in the valid space, then this information is used generate futures samples with higher probability of being in the valid space. This idea is graphically presented in the Fig. 4.

B. The Equilibrium Manifold

Since we are doing planning for robots interacting with a grasped object, we need to guarantee that those interactions are safe for both, the robot and the object. During interactions it is necessary to ensure that the contact forces remain between the minimum and maximum values allowed, and within the friction cone. As discussed in [17], the fine contact force management can be assured at control time, without affecting the performance of the object motion. This property of the system is exploited here to speed up the planning algorithm for closed kinematic chains. Here, in fact, the contact force values are admitted to vary in a certain range, as we are going to explain, also without the necessity of considering contact limits leaving this task to the controller.

For later use, let us define $p_\mathrm{h} \in \mathbb{R}^c$ as the vector describing the mutual configuration of the contact points on the robots and on the object to be manipulated. Considering the closed loop constraint, for planning purpose we should randomly choose joint configurations q such that $p_\mathrm{h} = 0$. Conversely, relaxing the closed loop constraint, it is possible to admit vector values such that $p_\mathrm{h} < \epsilon$. Considering virtual springs at the contacts, whose characteristics are described by the contact stiffness matrix $K_\mathrm{c} \in \mathbb{R}^{c \times c}$, the contact forces between the robot and the object can be described as $f_\mathrm{h} = K_\mathrm{c} p_\mathrm{h} \in \mathbb{R}^c$. It is worth stressing the fact that admissible values for the planner $f_\mathrm{h} < K_\mathrm{c}\epsilon$ can violate contact limits. As a consequence, also negative forces (within a certain limit defined by the vector ϵ) can be considered acceptable for the planner, as well as tangential forces out of friction cone limits.

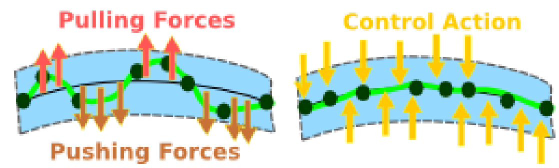
The equilibrium of the robot/object system can be assured if the following relationships are satisfied

$$
\begin{aligned}
w + G f_\mathrm{h} &= 0, && (2)\\
\tau - J^T(q,u) f_\mathrm{h} &= 0, && (3)\\
f_\mathrm{h} - K_\mathrm{c} p_\mathrm{h} &= 0, && (4)\\
\tau - K_\mathrm{q}(q_\mathrm{r} - q) &= 0, && (5)
\end{aligned}
$$

where $w \in \mathbb{R}^6$ is a possible external wrench acting on the object, $G \in \mathbb{R}^{6 \times c}$ is the *grasp matrix* of the system. The vector $u \in \mathbb{R}^6$ parametrize the configuration and the vector $\tau \in \mathbb{R}^d$ collects the joint torques. For the sake of generality, eq. (5) was introduced to consider the possibility of having also compliance at the joint level, where the joint stiffness matrix $K_\mathrm{q} \in \mathbb{R}^{d \times d}$ and the joint reference configuration vector $q_\mathrm{r} \in \mathbb{R}^d$ were used. The system of equations composed by (2), (3), (4) and (5), compactly written as $\Phi(\varphi) = 0 \in \mathbb{R}^{c+2d+6}$, where $\varphi \in \mathbb{R}^{c+3d+12}$ is a vector collecting all the system variables, describes the *equilibrium manifold* of the system. From recent results in grasp analysis, extensively discussed in [19] and in [20], it follows that it is valid to parametrize the equilibrium manifold with the variables w and q_r (or q if there is no elasticity at the joints). In other words, in case of no external wrench acting on the object, $w = 0$, the joint reference configuration is suficient to define the value also of q, u, τ, f_h and p_h. Given q_r all the other variables can be found solving the problem

$$\min_{q,u,f_\mathrm{h},\tau} \quad \Phi^T(\varphi)\Phi(\varphi). \qquad (6)$$

Once the variables u, q, τ and f_h are found, they can be used in the soft-RRT* to check for collisions and bounds of interaction forces.



(a) Undesired forces arising from planning on the relaxed contraint

(b) The task of the controller is to project the undesired forces back to the manifod

Fig. 5. Lateral view of the relaxed constraint. In green are the pushing and pulling forces against the constraint. The black dots are the nodes extracted from the tree generated by the $soft - RRT^*$.

V. Execution of the Planned Path

After the planning problem is solved a proper controller able to let the robot follow the planned path must be determined. The main challenge comes from the fact that the closed kinematic constraint has been relaxed, so undesired contact forces arise from interactions, a graphical example is shown in Fig. 5(a). The real-time controller must ensure that the nominal constraint is satisfied during the whole execution, see Fig. 5(b). Indeed, if only the relaxed constraint is verified, the object handled by the robot does not fall but can be damaged by possible high squeezing forces. On the other hand, whenever the nominal closed kinematic constraint is verified this can not occur.

The problem that arises when relaxing constraints is that from the point of view of implementation, the constraint violation can be dangerous, see Fig. 6, since undesirable interaction forces may be indirectly induced into the system. In this section we introduce a control law to overcome this problem.

A. Control

In order to address the problem of regulating contact forces and, at the same time, executing the planned path, a force/position controller can be implemented. There are many control approaches to do that, for example in [21] an adaptive hybrid control scheme for multiple geometric constraints based on the joint-space orthogonalization method (JSOM) is proposed, in [22] the authors propose a general

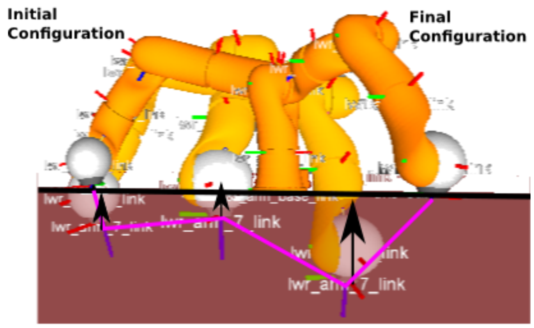


Fig. 6. In this example the Kuka robot has to move from the initial to the final configuration maintaining the contact with the plane (red object). The resulting path from the soft-RRT* is shown in pink and the forces during motions in black arrows. Forces that arise form the virtual violations of the contraint can damage either the robot mechanism or the environment in case of rigid robots. This issue is addressed by soft robots where compliance is included for safety.

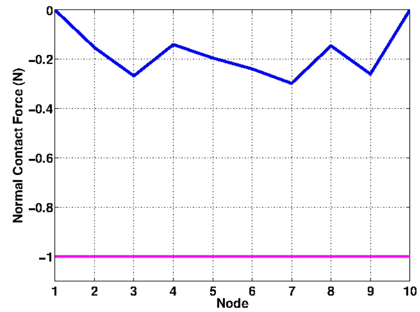


Fig. 9. Normal contact forces (blue) resulting from the relaxation of the constraint during planning. The maximum allowed forces ϵ are in magenta.

framework for multi-contact motion/force control. In both cases the main considerations is that contacts are performed with rigid environments. However, new robot developments, like Soft Robots, are designed to work in uncertain environments and compliant task spaces. A general analysis of manipulation systems with general kinematics and compliant contact models is presented in [17] and complemented in [23]. The main result of the last two contributions is a geometric description and an algorithm to provide a basis to describe the feasible motions that can be executed by the system, and forces that can be controlled to avoid violation of the contact constraints, both in a decoupled way. In practice it means that it is possible to control all object displacements given a fixed force reference and vice versa, where the first is useful to correct the relaxations in the planning phase.

VI. Simulations

In this section we show simulations results of the motion planning method presented in this paper. As an example we consider a two finger planar hand with two degrees of freedom in each finger, see Fig. 8.

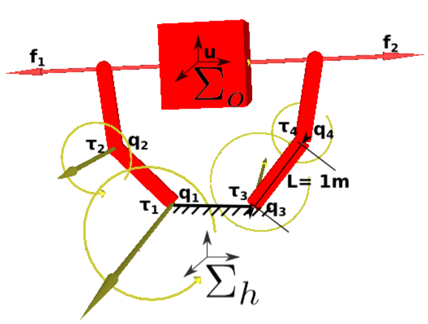


Fig. 8. The two finger hand used for the presented example. Object position are represented by u, the fingers configurations are $q = \{q_1, q_2, q_3, q_4\}$, the reaction torques are $\tau = \{\tau_1, \tau_2, \tau_3, \tau_4\}$ and contact forces are $f_h = \{f_1, f_2\}$.

This systems has 7 degrees of freedom in total, 2 joint positions for each robot and 3 to represent the position of the object in the space. For planning purposes we are just randomly sampling the joint positions, consequently the object positions are generated solving the equilibrium manifold equations. Algorithms have been implemented in C++ and use ROS for visualization purposes. All tests were performed in a $2.4Ghz$ quad-core computer with $3Gb$ of RAM memory and Ubuntu 14.04 operative system. Fig. 7(a) shows the starting position and Fig. 7(j) shows goal position of the hand, the objective is to find a path connecting this two points avoiding the static spherical obstacle in green and maintaining the contact forces within ϵ. Figs. 7(b) to 7(i) show some snapshots of the planned path resulting from the execution of algorithm 1, we can observe how the hand avoids the obstacle. Interaction forces arising from the planning phase, which in the case of the 2D example presented in this section are normal to the contact constraints and with magnitude proportional to ϵ, are shown in Fig. 9. Notice that the relaxation parameter ϵ is never overtaken.

VII. Conclusions

In this paper we propose a motion planning method for robots moving with task constraints. The approach consists in the combination of constraint relaxation and random sampling sub-optimal planner. The first one helps to speed up the planning phase considering the closed loop imposed in the system because of the interaction of the manipulators and the object. The second one allows us to explore the complete configuration space of the system and, at the same time, take into account optimality of the planned trajectories. Combining the first two strategies we are able to fast plan motions for multiple robot manipulators working cooperatively, however due to the constraint relaxation interaction forces appeared during executions of the planned path. To deal with this, as a future work we can implement a control strategy, as the ones presented in section V, to online regulate contact forces while executing trajectories coming from planning phase.

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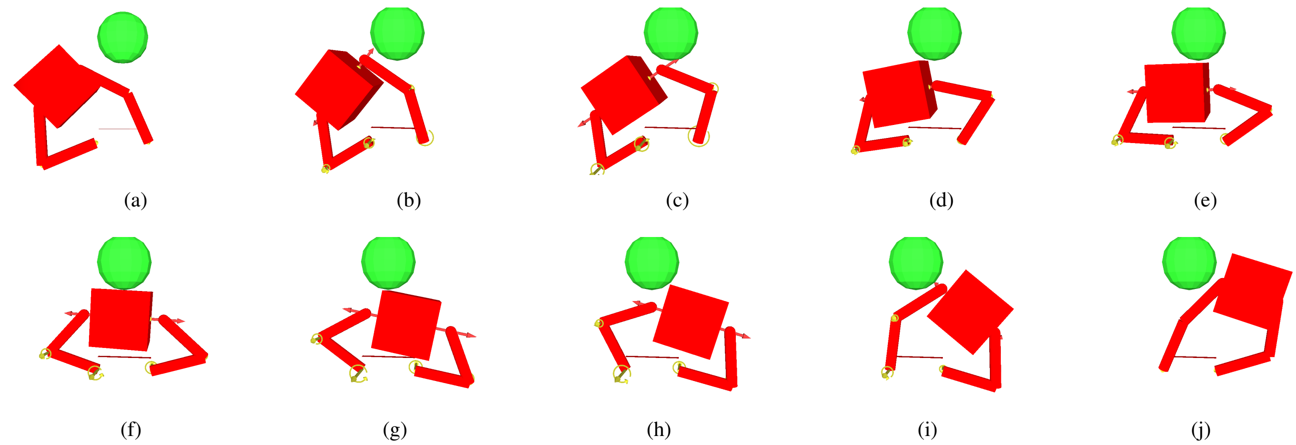

Fig. 7. Final path from the presented experiment. a) Initial position and j) final position.

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