On the Synthesis of Feasible and Prehensile Robotic Grasps

Carlos Rosales, Raúl Suárez, Marco Gabiccini, Antonio Bicchi

***Abstract*— This work proposes a solution to the grasp synthesis problem, which consist of finding the best hand configuration to grasp a given object for a specific manipulation task while satisfying all the necessary constraints. This problem is usually divided into sequential sub-problems, including contact region determination, hand inverse kinematics and force distribution, with the particular constraints of each step tackled independently. This may lead to unnecessary effort, such as when one of the problems has no solution given the output of the previous step as input. To overcome this issue, we present a kinestatic formulation of the grasp synthesis problem that introduces compliance both at the joints and the contacts. This provides a proper framework to synthesize a feasible and prehensile grasp by considering simultaneously the necessary grasping constraints, including contact reachability, object restraint, and force controllability. As a consequence, a solution of the proposed model results in a set of hand configurations that allows to execute the grasp using only a position controller. The approach is illustrated with experiments using a simple planar two-fingered hand and an anthropomorphic robotic hand.**

keywords: grasp synthesis, robotic hands, stiffness.

Fig. 1. Different multi-fingered robotic hand designs. The left image corresponds to THE first hand developed at the Centro E. Piaggio with five fingers, and the right image corresponds to the Schunk Anthropomorphic hand, based on the DLR II hand, with four fingers

I. Introduction

Mechanical anthropomorphic hands have been introduced for the last decades with the idea of performing dexterous manipulation tasks, that is, moving an object within the hand by means of finger motions [1]–[3]. Two different designs of a fingered robotic hand are shown in Fig. 1. In order to achieve this complex coordination autonomously, the hand must be previously commanded to grasp the object such that the subsequent manipulation can occur. Whether the task defines how the object should be grasped, or it is the grasping configuration that allow certain types of tasks is always a matter of fruitful discussions [4]. In both cases, the initial grasp configuration, the very first step in dexterous manipulation, is a crucial step to accomplish a given task. Finding such initial grasp configuration is called the grasp synthesis problem [1]. Although in most of the works dealing with dexterous manipulation using multi-fingered robotic hands it is assumed to be given [3, 5]–[9], a systematic and robust way to its definition is a wide open issue [4]. Solution to sub-problems exist forming a sequential approach. First, one seeks for the contact locations that restrain the object motions [10]–[12]. Second, one tries to find the hand configuration that reach to those locations [13]–[16], being the solution of the first problem used as input. Third, one selects the internal grasping forces given a change in the external force applied on the object, which assumes both that the first two problems were solved and the configuration is already in static equilibrium [17]–[22]. Recent research works have already started to merge some of the mentioned problems [23]–[26], however, they either miss some of the necessary constraints, limit the approach to planar grasps, or use a reduced number of fingers. A similar problematic is found as well on the synthesis of stable configurations of tensegrities [27, 28].

In this work, we propose a solution to the grasp synthesis problem using a kinestatic formulation. The approach is built upon an elastic model that introduces springs at the joints and the contacts modeling the compliances, which allows to tackle simultaneously the necessary constraints when synthesizing a feasible and prehensile grasp. These include the contact reachability, i.e. the contact points on the hand must touch the object surface adequately, the object restrainment, i.e. the object motion due to external perturbations is prevented by applying valid contact forces according to the contact model, and force controllability constraint, i.e. the valid contact forces must be compensated for entirely by joint torques and not by the structure. As a consequence, the solution results in a set of hand configurations that allow to execute the grasp using only a position controller, since the problem variables are configuration values.

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The paper is organized as follows: Section II presents the formulation of the feasible and prehensile grasp synthesis problem. Section III casts the formulation as an optimization problem using a potential energy-based criterion. Section IV shows the experimental results that validate the approach. Finally, Section V wraps up the the conclusions and remarks points deserving further study.

II. Kinestatic Formulation of the Grasp Synthesis Problem

The grasp synthesis problem consist of finding a feasible and prehensile grasp configuration. The formulation involves the specification of three hand configurations, as shown in Fig. 2. The outer hand configuration (black) accounts for the feasibility, i.e. the fingertips can actually touch the object surface given the kinematic structure of the fingers. The inner hand configuration (blue) is given as a reference produces joint torques that squeeze and restrain the object. The interaction produces reaction forces. Between them, the middle hand configuration (green) is the grasping configuration where the joint torques and contact forces are balanced accounting for prehensility, i.e. the system is in static equilibrium, whence the term kinestatic: kinematics + statics.

A. Model Description and Nomenclature

A grasp is a configuration of a hand and an object adjoined at certain contact points. Moreover, the preferred type of grasp for dexterous manipulation is the precision grasp, in which only one contact per finger at the fingertip is allowed [29], therefore, this is also adopted here.

A multi-fingered robotic hand is usually composed of several articulated fingers attached to a palm. The hand palm is positioned and oriented with respect to the world using the matrix $\mathbf{T}_H \in SE(3)$. The hand is composed of n fingers, each of them articulated through m_i revolute joints, for $i = 1, \ldots, n$, which sum up to $m = \sum_{i=1}^{n} m_i$ hand joints. The rotation angle of the j-th joint at the i-th finger is the joint value $q_{ij} \in \mathbb{S}$, where $\mathbb{S}$ denotes the angular nature of values. The phalanges are positioned and oriented with respect to the world using the homogeneous matrix $\mathbf{T}_{ij}$ depending on the joint values, q_{ij}, for $j = 1, \ldots, m_i$ and $i = 1, \ldots, n$. Thus, by collecting all joint values in the vector $\mathbf{q} = (q_{11}, \ldots q_{ij} \ldots, q_{nm_n})$, a configuration of the hand is represented by the pair $(\mathbf{q}, \mathbf{T}_H) \in \mathbb{S}^m \times SE(3)$.

There are n given contact points, one on each fingertip. A reference frame, $\mathbf{X}_c \in SE(3)$, is placed at the contact point $\mathbf{x}_c \in \mathbb{R}^3$, for $c = 1, \ldots, n$. The outward normal vector at the contact point is denoted as $\hat{\mathbf{n}}_c$. Both the contact point and the contact normal depend on the hand configuration $(\mathbf{q}, \mathbf{T}_H)$. The number of contact locations, n, is assumed to be, at least, the minimum required to restrain the object according to the chosen contact model (e.g. $n = 7$ for frictional contacts [1]).

The object is positioned and oriented with respect to the world using the matrix $\mathbf{T}_O \in SE(3)$. Without loss of generality, it is fixed and coincident with the world

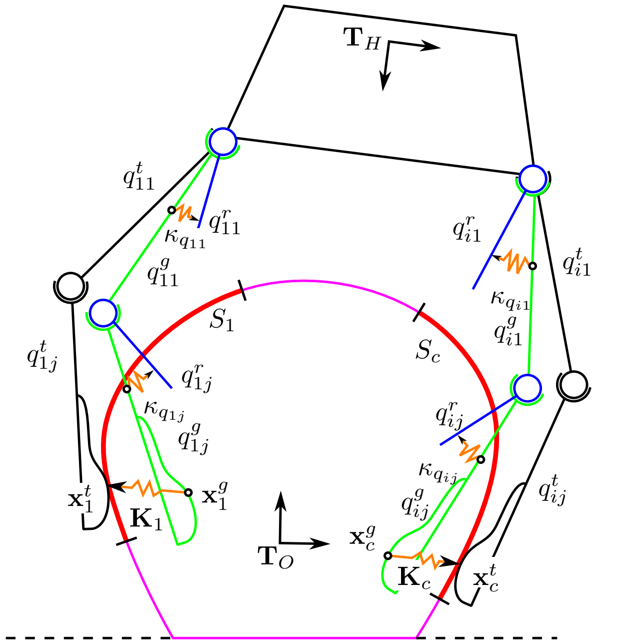

Fig. 2. Kinestatic model using springs at the joints and at the contact. The grasp is characterized by three hand configurations accounting for the feasiblity and prehensility of a grasp.

reference frame. On its surface, there is a given contact region, S_c, for each contact point, $\mathbf{x}_c$, on the hand. The coordinates of a point on the region S_c is obtained using the parametrization $\mathbf{s}_c(\boldsymbol{\gamma}_c)$, where $\boldsymbol{\gamma}_c \in \mathbb{R}^{D_c}$, with $D_c = 0, 1, 2$, the vector of parameters defining a point, a curve or a surface, respectively. The parametric inward normal vector at the point $\mathbf{s}_c$, is denoted as $\hat{\mathbf{m}}_c(\boldsymbol{\gamma}_c)$.

The contact forces and joint torques are modeled using a spatial spring at the contacts and a torsional spring at the joints with known stiffness constants $\mathbf{K}_c$ and $\kappa_{q_{ij}}$, respectively. The rest position for the contact springs is defined by the touching configuration of the hand, $(\mathbf{q}^t, \mathbf{T}_H)$, that makes the desired contact point, $\mathbf{x}^t_c$, on each finger reach the corresponding region, S_c, on the object. A reference configuration of the hand, $(\mathbf{q}^r, \mathbf{T}_H)$, pushes the fingers against the object surface loading the springs at the joints. The rest position of the joint springs is defined by the grasping configuration, $(\mathbf{q}^g, \mathbf{T}_H)$. Hence, the contact frame is moved to $\mathbf{X}^g_c$, which loads the springs at contact pushing the fingers towards $\mathbf{X}^g_c$.

B. Characterizing the Feasibility

A grasp is feasible when the touching configuration of the hand, $(\mathbf{q}^t, \mathbf{T}_h$ makes the points on the fingertips contact properly at the corresponding regions on the object, i.e. $\mathbf{x}^t_c \in S_c$. Thus, the contact reachability is written as

$$\|\mathbf{x}^t_c - \mathbf{s}_c(\boldsymbol{\gamma}_c)\|_2 = 0. \tag{1}$$

In order to properly position the fingertips on the object, the normal vectors at the contacting points are aligned by requiring

$$\hat{\mathbf{n}}^t_c \cdot \hat{\mathbf{m}}_c(\boldsymbol{\gamma}_c) = 1. \tag{2}$$

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TABLE I

BASIC NOTATION FOR KINESTATIC MODEL OF GRASP.

Sym.	Definition
$\mathbf{T}_h$	Hand reference frame
$\mathbf{T}_o$	Object reference frame
m	Number of joints
n	Number of fingers and contacts
q_{ij}	Value of the j-th joint at i-th finger
$\kappa_{q_{ij}}$	Stiffness of torsional spring of the j-th joint at the i-th finger
$\mathbf{q}^t$	Touching joint configuration
$\mathbf{q}^g$	Grasping joint configuration
$\mathbf{q}^r$	Reference joint configuration
$\delta\mathbf{q}$	Joint displacement from $\mathbf{q}^g$ to $\mathbf{q}^r$
$\mathbf{X}_c^g$	Contact reference frame, with origin at the point $\mathbf{x}_c^g$
$\mathbf{X}_c^t$	Contact reference frame, with origin at the point $\mathbf{x}_c^t$
$\delta\mathbf{X}_c$	The rigid body motion from $\mathbf{X}_c^t$ to $\mathbf{X}_c^g$
$\mathbf{K}_c$	Stiffness of the spatial spring at the c-th contact point
$\mathbf{w}_c$	Force due to the spatial spring at the c-th contact point
S_c	Contact region on the object corresponding to the c-th contact point
$\boldsymbol{\gamma}_c$	Parameter that defines a point on S_c
D_c	Dimensionality of region S_c (point, curve, surface)
$\mathbf{s}_c$	Contact point on the region S_c defined by $\boldsymbol{\gamma}_c$
$\hat{\mathbf{m}}_c$	Normal vector at the point $\mathbf{s}_c$ defined by $\boldsymbol{\gamma}_c$
$\mathbf{J}$	Hand jacobian evaluated at the grasping configuration
$\mathbf{G}$	Grasp matrix evaluated at the grasping configuration
$\mathbf{K}_q$	Joint stiffness matrix
$\mathbf{K}$	Contact stiffness matrix

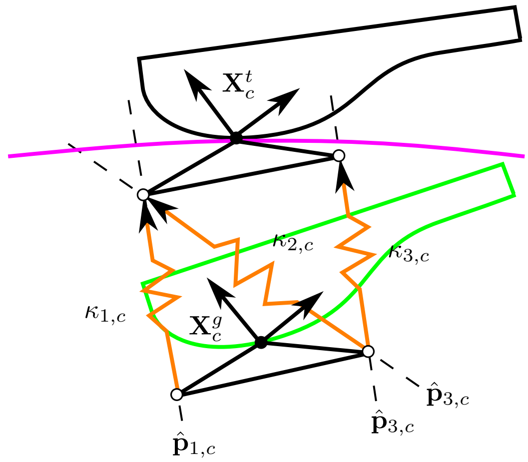

Fig. 3. Representation of the spatial spring placed at the contact locations. In this planar case, the contact model is SF, hence there are $h_c = 3$ linear springs. The triangles are drawn for clarity, however, the vertices coincident with the contact point, making $\mathbf{p}_{1,c}$, $\mathbf{p}_{2,c}$ and $\mathbf{p}_{3,c}$, pass through the origin of $\mathbf{X}_c$. The rest position of the springs correspond to the configuration where $\mathbf{X}_c^g = \mathbf{X}_c^t$.

Additionally, the position vector of the matrices $\mathbf{T}_{ij}$, $\mathbf{r}_{ij}$, are forced to be outside the object by requiring

$$\hat{\mathbf{m}}_c^T(\mathbf{s}_c - \mathbf{r}_{ij}) > 0, \quad (3)$$

which means that the projection of vector going from $\mathbf{r}_{ij}$ to $\mathbf{s}_c$ onto the normal at the contact point is positive.

Finally, the joint values of real robotic hands are subject to mechanical limitations. Hence, the touching configuration must fulfill the preceding constraints while the joint values stay within the valid range, that is,

$$\mathbf{q}^l \leq \mathbf{q}^t \leq \mathbf{q}^u, \quad (4)$$

with $\mathbf{q}^l$ and $\mathbf{q}^u$ the vectors of minimum and maximum values that they can reach, respectively.

C. Characterizing the Prehensility

The prehensility condition is met when the object motion due to external perturbations is prevented by applying valid contact forces according to the contact model due to the grasping configuration, also known as the object restrainment, and controlling those contact forces by applying joint torques due to the reference configuration, also known as force controllability. In the literature, they are also known as object equilibrium and hand equilibrium constraints, respectively [26]. It is worth noting that, the prehensility, together with the assumption that n is the minimum number of contacts required to restraint the object according to the contact model, yields a force-closure grasp as defined by [30].

Object Restrainment: Each contact force is modeled using a spatial springs conformed of h_c linear springs connecting the contact frames $\mathbf{X}_c^t$ and $\mathbf{X}_c^t$ (see Fig. 3). Thus, we express the effect of these springs acting on the object, i.e. the c-th contact force, as the sum of all spring forces as $\mathbf{w}_c = [\ \hat{\mathbf{p}}_{1,c} \ \ \dots \ \ \hat{\mathbf{p}}_{h_c,c}\ ]\boldsymbol{\lambda}_c$, where $\hat{\mathbf{p}}_{k,c} \in \mathbb{R}^6$ is the supporting line of the k-th spring passing through the contact point $\mathbf{x}_c^g$, and $\boldsymbol{\lambda}_c = [\ \lambda_{1,c} \ \ \dots \ \ \lambda_{h_c,c}\ ]^T$ collects the force magnitudes of the springs obtained as $\lambda_{k,c} = -\kappa_{k,c} d_{k,c}$, where $d_{k,c}$ is the spring elongation and $\kappa_{k,c}$ the stiffness constant of the k-th spring. Thus, the magnitude of the contact force can be written as $\boldsymbol{\lambda}_c = -\tilde{\mathbf{K}}_c \mathbf{d}_c$ using the diagonal matrix $\tilde{\mathbf{K}}_c = \text{diag}(\kappa_1, \dots, \kappa_{h_c})$ and $\mathbf{d}_c = [\ d_{1,c} \ \ \dots \ \ d_{h_c,c}\ ]^T$. Then, introducing the matrix $\mathbf{P}_c = [\ \hat{\mathbf{p}}_{1,c} \ \ \dots \ \ \hat{\mathbf{p}}_{h_c,c}\ ]$, we express the c-th contact force as

$$\mathbf{w}_c = -\mathbf{P}_c \tilde{\mathbf{K}}_c \mathbf{d}_c. \quad (5)$$

The displacement that goes from $\mathbf{X}_c^t$ to $\mathbf{X}_c^g$, $\delta\mathbf{X}_c \in SE(3)$, can be parametrized using six independent variables, known as the exponential coordinates [3], if it satisfies

$$\mathbf{X}_c^g = e^{(\delta\mathbf{X}_c)}\mathbf{X}_c^t, \quad (6)$$

where $e^{(\delta\mathbf{X}_c)}$ is the exponential map representing the relative finite rigid body displacement between them. The spring elongations is obtained by projecting the displacement onto the supporting lines of the springs as

$$\mathbf{d}_c = \mathbf{P}_c^T \delta\mathbf{X}_c. \quad (7)$$

Substituting (7) in (5) yields the expression of the contact force as a function of the touching and grasping configuration,

$$\mathbf{w}_c = -\mathbf{K}_c \delta\mathbf{X}_c, \quad (8)$$

where $\mathbf{K}_c = \mathbf{P}_c \tilde{\mathbf{K}}_c \mathbf{P}_c^T$.

Since we are assuming that n is the minimum number of contact points required, then the object restrainment is equivalent to the equilibrium of all contact forces, i.e. $\sum_{c=1}^{n} \mathbf{w}_c = 0$. Thus, building the ma-

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trix $\mathbf{G} = [\ \mathbf{P}_1 \ \ \dots \ \ \mathbf{P}_n\ ]$, the block diagonal matrix $\mathbf{K} = \text{blkdiag}(\tilde{\mathbf{K}}_1, \dots, \tilde{\mathbf{K}}_n)$, the block diagonal matrix $\mathbf{P} = \text{blkdiag}(\mathbf{P}_1, \dots, \mathbf{P}_n)$, and collecting all contact displacements in $\delta\mathbf{X} = [\ \delta\mathbf{X}_1^T \ \ \dots \ \ \delta\mathbf{X}_n^T\ ]^T$, the object restrainment can be expressed as

$$\mathbf{G}\mathbf{K}\mathbf{P}^T\delta\mathbf{X} = \mathbf{0}. \tag{9}$$

Additionally, the contact forces must comply with the contact model. Typical contact models in grasping include the point contact without friction (PC), point contact with friction (PCWF), and contact with a soft finger (SF) [1]. They can be implemented by considering springs only at the constrained directions, with $h_c = 1, 3, 4$, depending whether we use a PC, PCWF, or SF, respectively. Without loss of generality, the supporting lines of the springs are chosen such that $\hat{\mathbf{p}}_{1,c}$ indicates a translation along the inward normal direction, $\mathbf{p}_{2,c}$ and $\hat{\mathbf{p}}_{3,c}$ indicates translations along the tangent directions, and $\hat{\mathbf{p}}_{4,c}$ indicates a rotation about the normal direction. Thus, the linear and torsional friction coefficients, μ_c and ν_c, define an additional constraint on the vector $\mathbf{w}_c = [\ w_{1,c} \ \ \dots \ \ w_{6,c}\ ]$, which must belong to the generalized friction cone

$$\mathcal{C}_c = \{\mathbf{w}_c \in \mathbb{R}^6 | \|\mathbf{w}_c\|_\Delta \leq w_{1,c}\}, \tag{10}$$

where $\|\mathbf{w}_c\|_\Delta$ can take the form 0, $\frac{1}{\mu_c}\sqrt{w_{2,c}^2 + w_{3,c}^2}$ or $\frac{1}{\mu_c}\sqrt{w_{2,c}^2 + w_{3,c}^2} + \frac{1}{\nu_c}|w_{4,c}|$ depending on whether we use the PC, PCWF, or SF model, respectively.

Force Controllability: Each joint torque is modeled using a torsional spring connecting the grasping and the reference configuration at the joints. The resultant force due to the spring elongation is written as

$$\mathbf{w}_{ij} = \hat{\mathbf{z}}_{ij}\tau_{ij}, \tag{11}$$

where $\hat{\mathbf{z}}_{ij}$ is the supporting line that coincides with the joint rotation axis at the grasping configuration, and τ_{ij} is the torque magnitude obtained as $\tau_{ij} = \kappa_{ij}d_{ij}$, where d_{ij} is the spring elongation. The joint displacement of the i-th finger is expressed as $\delta\mathbf{q}_i = \mathbf{q}_i^r - \mathbf{q}_i^g$, where the subscript i indicates that only the m_i joints of the i-th finger are used. Thus, introducing the matrix $\mathbf{Z}_i = [\ \hat{\mathbf{z}}_{i,1}^T \ \ \dots \ \ \hat{\mathbf{z}}_{i,m_i}^T\ ]^T$, the joint torque magnitudes that result of applying a force $\mathbf{w}_c$ at the c th contact point is

$$\mathbf{K}_{q_i}\delta\mathbf{q_i} = \mathbf{Z}_i^T\mathbf{w}_c, \tag{12}$$

where $\mathbf{K}_{q_i} = \text{diag}(\kappa_{q_{i1}}, \dots, \kappa_{q_{im_i}})$, for $c = 1, \dots, n$ and $i = c$ in turns.

The controllability is achieved when the contact forces are compensated by joint torques. Since the fingers are independent, the force applied at the c-th should be compensated by torques at the i-th finger, with $i = c$. Thus, introducing the block diagonal matrix $\mathbf{J} = \text{blkdiag}(\mathbf{Z}_1, \dots, \mathbf{Z}_n)$ and the vector $\mathbf{w} = [\ \mathbf{w}_1^T \ \ \dots \ \ \mathbf{w}_n^T\ ]^T = \mathbf{P}\mathbf{K}\mathbf{P}^T\delta\mathbf{X}$, the hand force controllability can be expressed as

$$\mathbf{K}_q\delta\mathbf{q} = \mathbf{J}^T\mathbf{w}, \tag{13}$$

where $\mathbf{K}_q$ and $\delta\mathbf{q}$ consider all joints ordered accordingly to the corresponding row of $\mathbf{J}^T$.

In addition, the grasping configuration must be reached by the actual hand, hence the joint value limitations are again applicable here as

$$\mathbf{q}^l \leq \mathbf{q}^g \leq \mathbf{q}^u. \tag{14}$$

Finally, the joint torques are subject to mechanical limitations as well, written as

$$|\mathbf{K}_q\delta\mathbf{q}| \leq \boldsymbol{\tau}^{\max}, \tag{15}$$

with $\boldsymbol{\tau}^{max}$ the vector of maximum torque that the joint motors can exert, and the absolute value is done component-wise. Note that, $\mathbf{q}^r$ is not subject to the mechanical limitations, however, to guarantee that the fingers are pushing against the object, we must ensure that they are exerting a minimum torque $\boldsymbol{\tau}^{min}$ by including

$$|\mathbf{K}_{q_i}\delta\mathbf{q}_i| \leq \boldsymbol{\tau}^{\min}, \tag{16}$$

again, with the absolute value obtained component wise.

D. System Overview and Dimension Analysis

A grasp configuration $\mathbf{y} = (\mathbf{q}^r, \mathbf{q}^g, \mathbf{q}^t, \mathbf{T}_h, \gamma)$ is feasible and prehensile if it fulfills (1), (2), (9), and (13) collected in

$$\mathbf{M}_{\text{eq}}(\mathbf{y}) = \mathbf{0}, \tag{17}$$

and (3), (10), (15), and (16), transformed in less-than-equal inequalities and collected in

$$\mathbf{M}_{\text{ineq}}(\mathbf{y}) \leq \mathbf{0}, \tag{18}$$

while staying within the valid ranges defined by (4) and (14), for $c = 1, \dots, n$ contacts, $j = 1, \dots, m_i$ joints and $i = c$ (in turns) fingers.

The dimension of the solution space depends on the number of variables and the number algebraic constraints. The number of variables is $n_v = 3m + 6 + \sum_{c=1}^{n} D_c$ and the number of algebraic constraints is $n_e = 6n + m + 6$, i.e. the $3n$ contact point reachability equations, the $3n$ normal vector alignment, the m joint torque compensations and the 6 object restrainment equations. Assuming $D = D_c$, for $c = 1, \dots, n$, the solution space is then of dimension $n_s = n_v - n_e = 2m + n(D - 6)$. In general, this number is high when considering surfaces, i.e. $D = 2$.

III. Criterion for Uniqueness and Solution Strategy

There may be multiple solutions due to the high nonlinearities in the constraints, even when the solution space could be of dimension zero. Thus, we propose the potential energy of the springs at the joints as the criterion to select among the possible solutions expressed as

$$\Psi(\mathbf{y}) = \frac{1}{2}\delta\mathbf{q}^T\mathbf{K}_q\delta\mathbf{q}, \tag{19}$$

leading to hand configurations where $\mathbf{q}^t$, $\mathbf{q}^g$, and $\mathbf{q}^r$ are close to each other. It is worth noting that, when the constraints are satisfied, the substitution of (13) in (19), yields

$$\Psi'(\mathbf{y}) = \frac{1}{2}\delta\mathbf{q}^T(\mathbf{J}^T\mathbf{P}\mathbf{K}\mathbf{P}^T\delta\mathbf{X}). \quad (20)$$

and additionally, $\delta\mathbf{X} \approx \mathbf{J}(\mathbf{q}^g - \mathbf{q}^t)$. Thus, introducing the block diagonal matrix $\mathbf{K}' = \mathbf{P}\mathbf{K}\mathbf{P}^T$, (20) becomes

$$\Psi''(\mathbf{y}) = \frac{1}{2}(\mathbf{q}^r - \mathbf{q}^g)^T\mathbf{J}^T\mathbf{K}'\mathbf{J}(\mathbf{q}^g - \mathbf{q}^t). \quad (21)$$

Comparing (21) and (19), we can rewrite the criterion as

$$\Psi'''(\mathbf{y}) = \|\mathbf{K}_q - \mathbf{J}^T\mathbf{K}'\mathbf{J}\|, \quad (22)$$

i.e. the goal is to find a configuration $\mathbf{y}$ in which the joint stiffness, $\mathbf{K}_q$ is equivalent to the contact stiffness, $\mathbf{K}'$.

Now, the problem can be casted as: Given a hand with n articulated fingers, with a kinematic configuration defined by the pair $(\mathbf{q}, \mathbf{T}_h)$, contacts on the fingertips $\mathbf{X}_c$, corresponding contact regions on the object surface S_c, m joint spring stiffnesses κ_{ij} and nh_c contact spring stiffnesses $\kappa_{k,c}$, and friction coefficients μ_c and ν_c, find a configuration $\mathbf{y}$ that minimizes the objective function (22) subject to the constraints (17-18), (4), and (14). This non-linear optimization problem is in the form required by the MATLAB routine `fmincon`. We select the `SQP` algorithm due to its ability to work out of the solution manifold using a feasiblity reformulation. This slows down the process, however it is desired when the method is starting and the configurations are far from satisfying the constraints [31].

IV. Experiments and Results

The method is illustrated here with two experiments. The first one consists of a simple planar hand grasping an ellipse, and the second one, of a complex robotic hand grasping an ellipsoid.

Example 1. A simple planar hand grasping an ellipse: Here, we use a simple hand with $n = 2$ fingers, and $m_i = 2$ joints, for a total of $m = 4$ joints. The object is an ellipse, and the contact regions cover fully the ellipse boundary. The kinematic structure, spring constants and friction coefficients needed to write the grasp synthesis problem as stated in Section III are shown in Table II. In this case, the initial guess was randomly generated, but biased towards the mean value of the limit values of the variables. The results using the proposed method from two different initial guesses are shown in Fig. 4.

Example 2. An anthropomorphic hand grasping an ellipsoid: Here, we use the Schunk anthropomorphic hand shown at the right of Fig. 1. The grasp uses $n = 3$ fingers (three out of the four available), with $m_1 = 3$, $m_2 = 3$, and $m_3 = 4$ joints, for a total of $m = 10$. The object is an ellipsoid, and the contact regions cover fully its surface. The kinematic structure, spring constants and friction coefficients needed to write the grasp synthesis problem as stated in Section III are shown in Table II. In this case, the initial guess was set by introducing the constraints sequentially, such that it was as close as possible from the solution manifold. generated and biased towards the mean value of the limit values of the variables. The result using the proposed method is shown in Fig. 5.

TABLE II
PARAMETERS FOR EXAMPLE 1.

a) Kinematic structure and limit values	
Finger anchors	$T_{11} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,
	$T_{21} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Pahalanx length [cm]	All phalanges are of length 1
Joint limits [deg]	$\mathbf{q}^l = \begin{bmatrix} 0 \\ -90 \\ 0 \\ -90 \end{bmatrix}$, $\mathbf{q}^u = \begin{bmatrix} 90 \\ 45 \\ 90 \\ 45 \end{bmatrix}$
Torque limits [Ncm]	$\boldsymbol{\tau}^{\min} = 1[\mathbf{1}]$, $\boldsymbol{\tau}^{\max} = 10[\mathbf{1}]$, where $[\mathbf{1}] \in \mathbb{R}^m$ is a vector containing ones
Contact point $\mathbf{x}_c, \forall c$ in local [cm]	$\mathbf{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Regions $S_c, \forall c$ [cm]	$\mathbf{s} = \begin{bmatrix} 0.8\cos(\gamma_c) \\ 0.7\sin(\gamma_c) \end{bmatrix}$, with $\gamma_c \in [0, 2\pi]$
b) Coefficients	
Joint stiffness [N/rad]	$\mathbf{K}_q = \text{blkdiag}(1,1,1,1)$
Friction (PCWF) $\mu_c, \forall c$	$\mu = 0.5$
Contact stiffness $\tilde{\mathbf{K}}_c, \forall c$ [N/cm]	$\tilde{\mathbf{K}} = \text{blkdiag}(1,1)$

V. Conclusions and Future Work

The proposed approach tackles simultaneously the contact reachability, the object restrainment and the force controllability constraints that a feasible and prehensile grasp must satisfy. This is obtained by introducing torsional springs modeling the joint compliance, and spatial springs for the contact interaction. This leads to a solution where all variables ultimately employs configuration values, and therefore, the hand can be commanded to grasp the object using only a position controller.

The results show that the proposed method provides practicable solutions for a illustrative examples suggesting how to: (i) reach the specified regions on object with the fingertips, (ii) apply the forces in the directions allowed by the contact model within the friction constraints, and (iii) compensate such forces using the hand joints, i.e. the hand performs a feasible and prehensile grasp of the object with the minimum effort.

A potential simplification of the problem would be introducing the concept of postural synergies into the model, since coupling the hand joints in a human-like behavior reduces the problem dimensionality. Also, it would be worth studying the influence of considering contacts on the hand palm, as in certain human grasp situations.

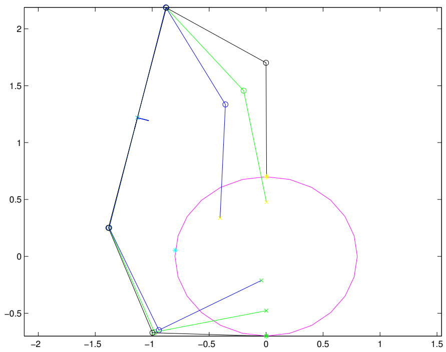
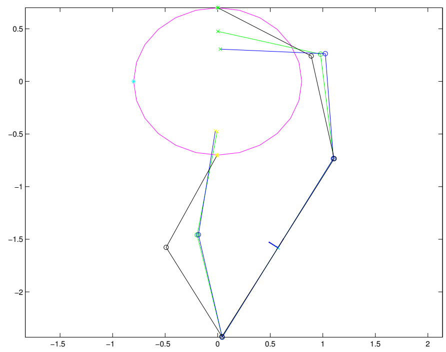

Fig. 4. Two solutions obtained for the simple hand satisfying all constraints.

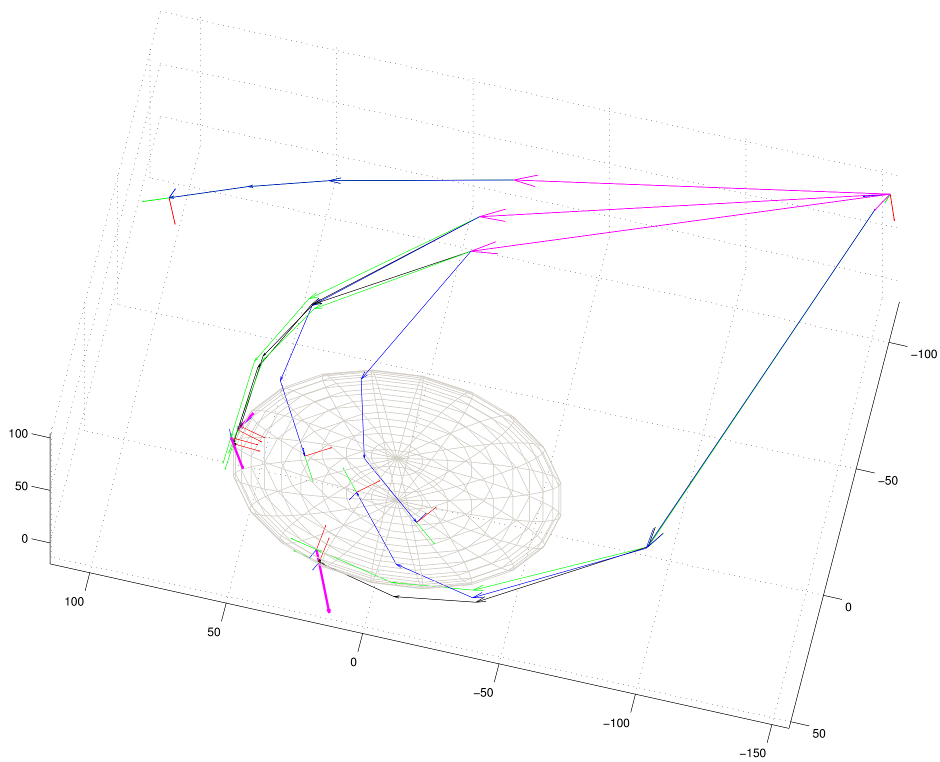

Fig. 5. A solution obtained for the Schunk Anthropomorphic hand satisfying all constraints performing the grasp with three fingers.

TABLE III
PARAMETERS FOR EXAMPLE 2.

a) Kinematic structure and limit values	
Finger anchors in local	$T_{11} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 27.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $T_{21} = \begin{bmatrix} 0 & 0 & 1 & -4.3 \\ 0.035 & -0.99 & 0 & 40.165 \\ 0.99 & 0.035 & 0 & 145.43 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $T_{31} = \begin{bmatrix} 0 & 0 & 1 & -4.3 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 145.43 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
DH parameters $(a_j, \alpha_j, d_j)_i$ [cm,rad,cm]	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -\pi/2 & 0 \\ 67.8 & 0 & 0 \\ 30 & 0 & 0 \end{pmatrix}$, for $j = 1, 2, 3, 4$, for $i = 1, 2, 3$, and $q_{i3} = q_{i4}$
Joint limits [deg]	$\mathbf{q}^l = \begin{bmatrix} 0 \\ -15 \\ -4 \\ 4 \\ -15 \\ -4 \\ 4 \\ -15 \\ -4 \\ 4 \end{bmatrix}$, $\mathbf{q}^u = \begin{bmatrix} 90 \\ 15 \\ 75 \\ 75 \\ 15 \\ 75 \\ 75 \\ 15 \\ 75 \\ 75 \end{bmatrix}$
Torque limits [Ncm]	$\boldsymbol{\tau}^{\min} = 10[\mathbf{1}]$, $\boldsymbol{\tau}^{\max} = 1000[\mathbf{1}]$, where $[\mathbf{1}] \in \mathbb{R}^m$ is a vector containing ones
Contact point $\mathbf{x}_c, \forall c$ in local [cm]	$\mathbf{x} = \begin{bmatrix} 0 \\ 29.5 \\ 0 \end{bmatrix}$
Regions $S_c, \forall c$ [cm]	$\mathbf{s} = \begin{bmatrix} 60\cos(\gamma_{1,c})\sin(\gamma_{2,c}) \\ 40\sin(\gamma_{1,c})\sin(\gamma_{2,c}) \\ 20\cos(\gamma 2, c) \end{bmatrix}$, with $\gamma_{1,c} \in [0, 2\pi]$ and $\gamma_{2,c} \in [0, \pi]$
b) Coefficients	
Joint stiffness [N/rad]	$\mathbf{K}_q = 100[\text{blkdiag}(4, 3, 2, 1, 3, 2, 1, 3, 2, 1)]$
Friction (PCWF) $\mu_c, \forall c$	$\mu = 0.25$
Contact stiffness $\tilde{\mathbf{K}}_c, \forall c$ [N/cm]	$\tilde{\mathbf{K}} = \text{blkdiag}(5, 5, 5)$

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