

Exploiting Packet Size in Uncertain Nonlinear Networked Control Systems [★]

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Abstract

This paper addresses the problem of stabilizing uncertain nonlinear plants over a shared limited-bandwidth packet-switching network. While conventional control loops are designed to work with circuit-switching networks, where dedicated communication channels provide almost constant bit rate and delay, many networks, such as Ethernet, organize data transmission in packets, carrying larger amount of information at less predictable rates. We adopt a model-based approach to remotely compute a predictive control signal on a suitable time horizon. By exploiting the inherent packets payload, this technique effectively reduces the bandwidth required to guarantee stability. Communications are assumed to be ruled by a rather general protocol model, which encompasses many protocols used in practice. An explicit bound on the combined effects of the maximum time between consecutive accesses to each node (MATI) and the transmission and processing delays (MAD), for both measurements and control packets, is provided as a function of the basin of attraction and the model accuracy. Our control strategy is shown to be robust with respect to sector-bounded uncertainties in the plant model. Sampling of the control signal is also explicitly taken into account. A case study is presented which enlightens the great improvements induced by the packet-based control strategy over methods that send a single control value in each packet.

1 Introduction

Industrial manufacturing is witnessing an ever more extensive use of communication networks to support automated scheduling, control and diagnostic activities [16], [37]. The possibility offered by networks of replacing traditional point-to-point connections with more complex and dynamic schemes, opens unprecedented opportunities for factory control and management. Alongside allowing a pervasive adoption of decentralization and cooperation, networks convey many advantages in terms of flexibility, scalability and robustness. The adoption of a distributed networked architecture can induce a remarkable reduction of costs and delays for both installation and maintenance. These advantages justify the increas-

ing interest in control over networks (see for instance [5], [1], [2], [7], [33]).

In general terms, a Networked Control System (NCS) is a system in which sensors, actuators and controllers are spatially distributed and exchange information through a shared, digital, finite capacity channel. The use of the network as a communication medium and the distributed nature of the system make traditional control theory not always applicable. Issues such as quantization errors, data dropouts, variable transmission intervals, variable communication delays, and constrained access to the network, can no longer be ignored [11]. The NCS literature has separately addressed many of these problems, and sometimes the combinations thereof. An excellent discussion of the state-of-the-art is reported in [10]. An essential aspect of NCS, not thoroughly analyzed in [10], is the packet-switching nature of many networks. As opposed to conventional control loops, which are designed to work with circuit-switching networks where dedicated communication channels provide almost constant bit rate and delay, networks such as Ethernet organize data transmission in packets, carrying larger amount of information at less predictable rates.

The organization of control information in data packets, which have relatively large transmission overhead,

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substantially alter the bandwidth/performance trade-off of traditional design. For instance, important data-rate theorems [12], [19], [20] expressing a fundamental relationship between the degree of instability of a given physical system and the minimum bit rate required to stabilize it, do not account for the fact that data come in packets with a minimum size (e.g. 84 bytes in Ethernet). To simplify, transmitting a 16 bits record every millisecond requires as much bandwidth in average as sending a packet of 84 bytes every 42 milliseconds; however, the implications on the effective sampling rate and feedback control performance are apparent. How to recover part of this performance is an objective of this study.

A second aspect inherent to packet-switching networks is transmission overhead. For instance, every Ethernet packet carries 38 bytes of headers and interframe separations, and useless information is necessarily padded into the payload to reach the minimum required packet length. As a consequence, transmitting a few bits per packet has essentially the same bandwidth cost as transmitting hundreds of them. A new, specific trade-off hence arises between packet rate and packet dimension for a given estimation/control task.

While the above aspects have been observed and described in the early literature on NCS (see e.g. the surveys [31], [14], [11]), only recently have appeared results which address them explicitly in controller design. The goal can be succinctly described as to decrease the network utilization (in terms of bandwidth, or packets per unit of time) without compromising control performance. In [15] a controller directly connected to the plant is considered and the number of measurement packets sent through the network is reduced by means of a state estimate provided by a model of the plant. In [3] the author pioneered the idea of sending feedforward control sequences, computed in advance on the basis of a model-based predictive (MBP) scheme, to the aim of compensating large delays in communication channels. A similar MBP scheme is exploited in [26], [27] and [25] to counteract packet dropouts in the controller-to-plant channel. Compensation of delays and packet dropouts in nonlinear NCSs is the main concern also of the MBP controllers developed in [17], [8]. Following developments along these lines generalized the technique to address time-varying delays and transfer intervals [24], some robustness problems with respect to bounded perturbations [23], as well as the constraints imposed by communication protocols on state measurement access [6].

In this paper we present a control strategy for packet-switching networks ensuring the stability of an uncertain nonlinear NCS affected by varying transmission intervals, varying (and potentially large) delays, and constrained access to the network. Building upon our early results in [6], we adopt the feedforward approach to send in a packet not only the control value to be applied at a specific instant, but also a prediction of the control law valid on a given time-horizon, so as to better exploit

the payload. In the same spirit of other model-based approaches (e.g. [15], [26], [24], [23]), the control sequence is obtained by simulating an (imprecise) model of the closed-loop plant. The internal state of the model is asynchronously updated by means of the measurements of the plant state provided by sensors. Due to their spatial distribution, only portions of the model state can be updated in each instant. Therefore, we consider the constrained access to the network to be ruled by a protocol deciding which sensor can communicate at each instant. The large control-packet, sent by the remote controller, is stored in an embedded memory on the plant side. Based on a local re-synchronization, made possible by a time-stamping of measurements, this strategy also allows to compensate the effect of bounded communication delays in the control loop. Unlike the commonly assumed small-delays hypothesis (see for instance [10]), we can compensate for delays larger than the transmission interval. We build our model upon the powerful hybrid formalism introduced in [21], and we consider network imperfections affecting both sides of the control loop. We provide explicit bounds on the Maximum Allowable Delay (MAD [10]) and on the Maximum Allowable Transfer Interval (MATI [32], i.e. the maximum duration between two successive communications) ensuring the exponential stability of the NCS over a prescribed basin of attraction. Finally, we clearly show, by means of a case study, the great improvement over existing methods that our feedforward control strategy induces on the aforementioned bounds.

A line of work close to ours is reported in [24], where the problem of stabilizing a nonlinear NCS with feedforward control sequences is addressed. Such sequences are computed by means of an approximate discrete-time plant model. The authors assume that the approximation algorithm is the only source of uncertainty in the model and that the inaccuracy of such a model can be reduced at will in order to achieve the desired MATI. In this paper, instead, we consider a robustness problem, where the plant uncertainty is given, and we provide a bound on the MATI in terms of the model inaccuracy (measured through its local Lipschitz constant).

2 Problem Statement

Notation: Given a set $A \subset \mathbb{R}$ and $a \in A$, $A_{\geq a}$ denotes the set $\{s \in A \mid s \geq a\}$. Given a vector $x = (x_1, \dots, x_n)^T \in \mathbb{R}^n$, $n \in \mathbb{N}_{\geq 1}$, $|x|$ denotes its Euclidean norm, i.e. $|x| \triangleq (\sum_{i=1}^n x_i^2)^{1/2}$. Given $R \geq 0$, B_R denotes the closed ball of radius R centered in zero: $B_R \triangleq \{x \in \mathbb{R}^n \mid |x| \leq R\}$. Given a locally essentially bounded signal $u : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$, $\|u\|_{\mathcal{L}_{\infty}} \triangleq \text{ess sup}_{t \geq 0} |u(t)|$. We use mod to denote the modulo operator, i.e. given $m, n \in \mathbb{N}$, $m \bmod n = p$ if and only if there exists $r \in \mathbb{N}$ such that $m = rn + p$ with $p < n$. We define the floor function $\lfloor \cdot \rfloor : \mathbb{R} \rightarrow \mathbb{Z}$ as $\lfloor x \rfloor \triangleq \max\{m \in \mathbb{Z} \mid m \leq x\}$.

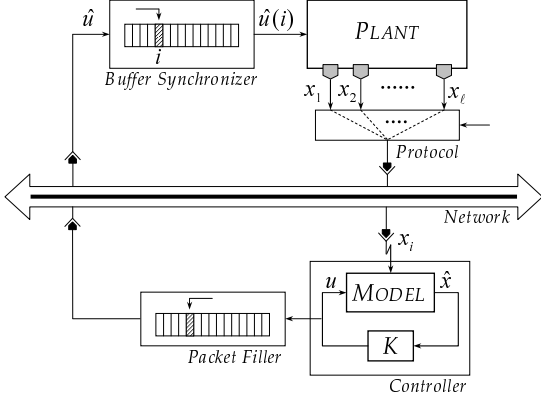


Fig. 1. Networked Control System with packet-switching network and protocol.

2.1 Network Model

We consider a NCS constituted of a remote controller receiving measurements from and sending commands to a physical plant through a shared communication channel (see Figure 1). Control sequences are sent over the digital network as packets. An elementary embedded control device receives, decodes, synchronizes these packets (see Buffer Synchronizer in Figure 1) and applies control commands to the plant. Measurements are taken by physically distributed sensors and sent towards the controller as packets encoded with sufficient precision to neglect quantization effects. Sensors are assumed to be embedded with the plant and hence synchronized with it. Due to the distributed nature of the network is partitioned in ℓ nodes and only a *unique* node at a time can send its information (i.e. only partial knowledge of the plant state is available at each time instants). In other words, the state $x \in \mathbb{R}^n$, $n \in \mathbb{N}_{\geq 1}$, of the plant is decomposed as $x = (x_1^T, \dots, x_\ell^T)^T$ with $x_i \in \mathbb{R}^{p_i}$ and $\sum_{i=1}^{\ell} p_i = n$.

We consider that measurements are taken and sent at instants $\{\tau_i^m\}_{i \in \mathbb{N}}$, and are received by the remote controller at instants $\{\tau_i^m + T_i^m\}_{i \in \mathbb{N}}$. In other words, $\{T_i^m\}_{i \in \mathbb{N}}$ denote the (possibly time-varying) measurement data delays, which cover both processing and transmission delays on the measurement chain. In the same way, control commands are computed, encoded into packets (see Packet Filler in Figure 1) and sent over the network at time instants $\{\tau_j^c\}_{j \in \mathbb{N}}$. They reach the plant at instants $\{\tau_j^c + T_j^c\}_{j \in \mathbb{N}}$, where $\{T_j^c\}_{j \in \mathbb{N}}$ denote the (possibly time-varying) control data delays accounting both for computation and transmission delays from the remote controller to the plant.

Assumption 1 (Network) *The communication network satisfies the following properties:*

i) (**MATI**) *There exist two constants $\tau_m, \tau_c \geq 0$ such that $\tau_{i+1}^m - \tau_i^m \leq \tau_m$ and $\tau_{j+1}^c - \tau_j^c \leq \tau_c$, $\forall i, j \in \mathbb{N}$;*

ii) (**MAD**) *There exist two constants $T_m, T_c \geq 0$ such that $T_i^m \leq T_m$ and $T_j^c \leq T_c$, $\forall i, j \in \mathbb{N}$;*

iii) (**mTI**) *There exist constants $\varepsilon_m > 0$ and $\varepsilon_c > 0$ such that $\varepsilon_m \leq \tau_{i+1}^m - \tau_i^m$, $\forall i \in \mathbb{N}$ and $\varepsilon_c \leq \tau_{j+1}^c - \tau_j^c$, $\forall j \in \mathbb{N}$.*

Item i) in the previous assumption imposes that the time between two consecutive accesses to the network is bounded both for measurements and control. Item ii) imposes that the maximum delay (MAD), both on measurements and control side, is bounded. Item iii) imposes that the minimum time interval (mTI) between two consecutive accesses to the network by the nodes is lower bounded away from zero, and similarly for the control side. This assumption prevents Zeno phenomena to occur. The objective of this paper is to provide explicit bounds on the MATIs (τ_m and τ_c) and on the MADs (T_m and T_c) to guarantee exponential stability of the closed-loop NCS based on a specific control procedure.

2.2 Protocol Model

The access to the network is ruled by a protocol choosing, at each instant τ_i^m , which node communicates its data. Decisions can be taken either according to the time index i (static protocol) or based on the value of the network induced error e between the state estimate \hat{x} and the system state x (dynamic protocol). More precisely, in the spirit of [21], we model the network protocol as a time-varying discrete-time system involving the error $\mathbb{R}^n \ni e = (e_1^T, \dots, e_\ell^T)^T \triangleq \hat{x} - x$, that this type of communication generates:

$$e(i+1) = h(i, e(i)), \quad \forall i \in \mathbb{N}, \quad (1)$$

where $h : \mathbb{N} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$. It is worth noting that, even if the function h in (1) explicitly depends on the whole error e , a centralized knowledge of e is not required. In fact, a distributed implementation of the protocol is possible since any measurement node $r \in \{1, \dots, \ell\}$ is aware of the actual value x_r of the portion of system state it aims at transmitting and, hence, it is aware of the error e_r . In [31], for instance, the low level arbitration mechanism provided by the CAN infrastructure is exploited to grant access to the node r exposing the maximum error e_r , thus realizing a distributed protocol.

If the network were able to send the measurement of the whole state at each time instant τ_i^m , then the function h would be identically zero; this is an assumption commonly posed in the literature on NCSs (see for instance [4], [36], [15], [28], [35], [18], [26], [24], [23]) where network effects are mostly modeled as sampling and delays, but it may no longer be justified when sensors are physically distributed.

Purely static protocols involve a function h which takes as an argument the time index i only. An example of such protocols is the Round Robin (RR) protocol, which

executes a cyclic inspection of each node. On the opposite, some network protocols purely rely on the current value of the error, in which case h is independent of i : this is the case of the Maximum-Error-First Try-Once-Discard (MEF-TOD) protocol [32].

According to (1), when the protocol is dynamic, the knowledge of the error $e(\tau_i^m)$ at the transmission instant τ_i^m is required. Rigorously speaking, the sensors are not aware of the present value of the estimate \hat{x} : they just know the latest value of the measurements they have transmitted. This is not a problem as long as the system dynamics is slow with respect to the transmission intervals, but it could require very frequent transmissions. A way to address this issue is to adopt a solution similar to the one used in [34] for the definition of the CP-TOD protocol. Each sensor node can estimate the error value by means of the worst case growth rate of the system and can trigger the sending of measurements accordingly.

The objective of most communication protocols is to decrease some function of the network induced error e at each transmitted packet. A particularly relevant class of such protocols is the one that ensures an exponential decay of this error. We recall here a slightly modified version¹ of the definition in [21] to focus on the class of protocols we deal with in this work.

Assumption 2 (UGES Protocol) *The protocol modeled by the discrete-time system (1) is uniformly globally exponentially stable (UGES) and admits an associated Lyapunov function with bounded gradient. That is, there exist a function $W_0 : \mathbb{N} \times \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ locally Lipschitz in the second argument, and constants $\underline{\alpha}, \bar{\alpha}, c > 0$ and $\rho_0 \in [0, 1)$ such that, for all $e \in \mathbb{R}^n$ and all $i \in \mathbb{N}$,*

$$\underline{\alpha}|e| \leq W_0(i, e) \leq \bar{\alpha}|e| \quad (2)$$

$$W_0(i+1, h(i, e)) \leq \rho_0 W_0(i, e), \quad (3)$$

and for almost all $e \in \mathbb{R}^n$ and all $i \in \mathbb{N}$

$$\left| \frac{\partial W_0}{\partial e}(i, e) \right| \leq c. \quad (4)$$

It is worth stressing that the UGES protocols considered here are not necessarily *invariably* UGES, as assumed in [6]. Indeed, invariably UGES protocols require that the exponential convergence of the discrete update law of e implied by (3) is valid even when the update is performed according to an arbitrary time subsequence rather than at each step $i \in \mathbb{N}$. The latter property is rather restrictive, as it excludes, for instance, the commonly adopted Round Robin protocol.

¹ Our class of protocols adds condition (4) to the definition in [21]. This is equivalent at requiring W to be globally Lipschitz in e uniformly in i . As pointed out in [21] this condition is easily verified in many interesting cases (e.g. RR and MEF-TOD), thus not remarkably narrowing the class of useful protocols analyzed.

Remark 1 *We do not explicitly consider packet dropouts here. However, its inclusion is possible without modifying the overall framework if some additional assumptions are made. Dropouts in a plant-to-controller channel governed by an invariably UGES protocol (see [6]) are easily dealt with by considering a scaled MATI, as proposed in Remark II.4 in [10]. It should be noticed however that the MATI scaling approach does not apply if an UGES, but not invariably UGES, protocol is considered. Bounded packet dropouts in the controller-to-plant channel can be tolerated in our framework if consecutive feedforward control packets overlap sufficiently (this will be clearer in the sequel).*

Remark 2 *The mathematical formalism so far presented allows measurement and control packets to be sent according to two independent time sequences (i.e. τ_i^m and τ_j^c respectively). Such a model is then capable of encompassing both the case of two distinct channels for measurements and controls and the case of a unique network endowed with a protocol distinguishing between control and measurement packets. The only constraint on such a protocol is its ability to ensure that a control packet is sent at most every τ_c seconds. Measurement packets are sent in the temporal slots between each two sending of control packets and they are managed according to the UGES protocol described in Assumption 2. Hence, this protocol is placed on top of the UGES one used for measurements and can be implemented, for instance, by assigning a higher priority to control packets (see [30] for an implementation of the control packet priority in CAN bus).*

2.3 The plant and its model

We assume that a nominal static feedback controller is given, which would be able, in the absence of the effects induced by the network, to globally exponentially stabilize the time-invariant plant. More precisely, we assume the following.

Assumption 3 (Nominal GES) *There exists a continuously differentiable function $\kappa : \mathbb{R}^n \rightarrow \mathbb{R}^m$ such that the closed-loop system*

$$\dot{x} = f(x, u) \quad (5)$$

$$u = \kappa(x) \quad (6)$$

is globally exponentially stable (GES), so that there exists a differentiable function $V : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ and constants $\underline{\alpha}, \bar{\alpha}, \alpha, d > 0$ such that the following conditions hold for all $x \in \mathbb{R}^n$

$$\begin{aligned} \underline{\alpha}|x|^2 &\leq V(x) \leq \bar{\alpha}|x|^2 \\ \frac{\partial V}{\partial x}(x)f(x, \kappa(x)) &\leq -\alpha|x|^2 \\ \left| \frac{\partial V}{\partial x}(x) \right| &\leq d|x|. \end{aligned}$$

In order to compute the control signal, the remote controller makes use of a state estimate based on an approximate model \hat{f} of the plant f . Both the plant and its model are considered to be zero at the origin ($f(0, \kappa(0)) = \hat{f}(0, \kappa(0)) = 0$). The strategy developed in this paper relies on the assumption that the plant, its model and the nominal controller are all locally Lipschitz.

Assumption 4 (Local Lipschitz) *Given some constants $R_x, R_u > 0$, there exist some positive constants λ_f and λ_κ ² such that for all $x_1, x_2 \in B_{R_x}$ and all $u_1, u_2 \in B_{R_u}$, the following inequalities hold*

$$|f(x_1, u_1) - f(x_2, u_2)| \leq \lambda_f (|x_1 - x_2| + |u_1 - u_2|) \quad (7)$$

$$|\kappa(x_1) - \kappa(x_2)| \leq \lambda_\kappa |x_1 - x_2|. \quad (8)$$

It is worth noting that the previous assumption represents a further important relaxation with respect to [6], where *globally* Lipschitz vector fields were assumed. Finally, we assume that the plant model inaccuracy is sector-bounded.

Assumption 5 (Sector-Bounded Model Inaccuracy) *Given $R_x, R_u > 0$, there exists a nonnegative constant $\lambda_{f\hat{f}}$ such that, for all $x \in B_{R_x}$ and all $u \in B_{R_u}$,*

$$\left| \hat{f}(x, u) - f(x, u) \right| \leq \lambda_{f\hat{f}} (|x| + |u|). \quad (9)$$

The constant $\lambda_{f\hat{f}}$ thus measures the model accuracy: the closer the model \hat{f} is to the real system f , the smaller is $\lambda_{f\hat{f}}$ (in the ideal case of perfect modeling, it would be zero). Note that Assumption 5 allows to cope with both parametric uncertainties and unmodeled dynamics.

3 A model-based strategy

3.1 Modeling the overall setup

We develop here a model-based strategy exploiting the relatively large payload of a packet. At each reception of a new measurement, the remote controller updates an estimate of the current state of the plant and computes a prediction of the control signal over a fixed time horizon T_0^p by numerically running the model \hat{f} . This signal is then coded, marked with the time stamp of the measurement used to build it, and sent in a single packet at the next network access (see Packet Filler in Figure 1). When received by the plant, it is decoded and re-synchronized by the embedded computer (on board

of the plant). In particular, the embedded computer compares the packet time stamp with its internal clock and chooses the right starting point in the control sequence, namely the point corresponding to the plant's present time (see Buffer Synchronizer in Figure 1). This way, bounded communication delays in the control loop can be compensated, modulo the plant model inaccuracy. In order for the measurement time stamp to be used for the re-synchronization of the control sequence, we assume that the embedded computer, the plant and its sensors have a common clock. On the other hand, we stress that in our strategy there is no need for clock synchronization between the plant and the remote controller, as the latter exploits the time stamp received with the measurements to mark the control sequence. For sake of mathematical rigor, we introduce first a model accounting for infinitely many state variables and infinitely many duplicates of the model \hat{f} . In Section 3.2, we show how to properly reduce them to a finite number. For any measurement taken at τ_i^m , $i \in \mathbb{N}$, we consider a new estimate state variable $\hat{x}_i = (\hat{x}_i^{(1)T}, \dots, \hat{x}_i^{(\ell)T})^T$, valid over the time interval $[\tau_i^m, \tau_i^m + T_0^p]$, whose evolution is given by

$$\begin{aligned} \dot{\hat{x}}_i(t) &= \hat{f}(\hat{x}_i(t), \kappa(\hat{x}_i(t))), \quad \forall t \in [\tau_i^m, \tau_i^m + T_0^p] \\ \hat{x}_i(\tau_i^{m+}) &= x(\tau_i^m) + h(i, \hat{x}_{i-1}(\tau_i^m) - x(\tau_i^m)). \end{aligned} \quad (10)$$

An explicit bound of the prediction horizon T_0^p in terms of the measurement and control MATIs and MADs is given in (13). It is important to remark that the dynamics (10) actually evolves in a virtual (simulated) time. Indeed, a new measurement (taken at τ_i^m) reaches the controller only at $\tau_i^m + T_i^m$ and then triggers the simulation of the dynamics (10) for a virtual time interval $[\tau_i^m, \tau_i^m + T_0^p]$. The actual time spent for this simulation and for the computation of the predicted control signal is, in fact, included in the delay T_j^c . The dynamics in (10) is written as if it ran in real time, concurrently with the plant. This notation trick allows us to cast the overall system in a compact model similar to the one in [21]. The update of the estimate \hat{x}_i in (10) is equivalent to the error update in (1), thus it does not require the actual knowledge of the full state $x(\tau_i^m)$. Let us assume, for instance, that at τ_i^m the r -th node is scheduled to send its measurement $x_r(\tau_i^m)$. A very simple protocol can use such measurement to update the r -th component of \hat{x}_i without changing the other components

$$\begin{aligned} \hat{x}_i^{(r)}(\tau_i^{m+}) &= x_r(\tau_i^m) \\ \hat{x}_i^{(s)}(\tau_i^{m+}) &= \hat{x}_{i-1}^{(s)}(\tau_i^m), \quad \forall s \in \{1, \dots, \ell\} \setminus \{r\}. \end{aligned} \quad (11)$$

The previous protocol can also be described in terms of the function h as follows

$$\begin{aligned} h_r(i, e(\tau_i^m)) &= 0 \\ h_s(i, e(\tau_i^m)) &= -x_s(\tau_i^m) + \hat{x}_{i-1}^{(s)}(\tau_i^m) = e_s(\tau_i^m), \end{aligned}$$

² We stress that λ_κ can be chosen independently of R_u .

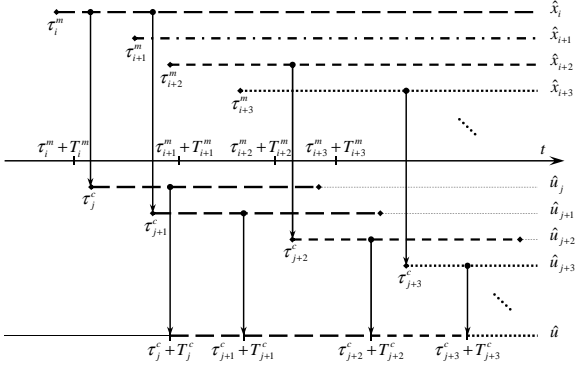


Fig. 2. Excerpt of an infinite sequence of estimate variables and control feedforward signals along with the control signal applied to the plant.

for every $s \in \{1, \dots, \ell\} \setminus \{r\}$, where $h_s(\cdot, \cdot)$ is the s -th component of the vector function h . Notice that, in our model a new estimate variable \hat{x}_i is created at each τ_i^m , with the previous variable \hat{x}_{i-1} containing the latest value of the estimate. Therefore, the update (10), and the example update (11), is performed in terms of the previous estimate variable $\hat{x}_{i-1}(\tau_i^m)$. In this way all measurements are used to continuously update the internal model.

The infinite sequence of evolutions for the simulated dynamics (10) is schematically depicted at the top of Figure 2, above the time line. Each simulated evolution is represented by a straight line starting at times τ_i^m , $i \in \mathbb{N}$ (explicitly reported at their left). Different line styles represent different evolutions for the estimate variables. The time line reports the instants $\tau_i^m + T_i^m$, $i \in \mathbb{N}$ at which the measurements reach the controller.

At each instant τ_j^c a new control signal $u_j(\cdot)$ is computed. It is based on the estimate variable $\hat{x}_{\gamma(j)}$, where $\gamma(j)$ denotes the index of the latest measurement received before τ_j^c . More precisely, the function $\gamma : \mathbb{N} \rightarrow \mathbb{N}$ is defined as

$$\gamma(j) \triangleq \max \{i \in \mathbb{N} \mid \tau_i^m + T_i^m < \tau_j^c\}, \quad \forall j \in \mathbb{N}. \quad (12)$$

In order to guarantee a sufficiently long prediction horizon T_0^p we consider the largest (worst case) time interval between the measurement taken at $\tau_{\gamma(j)}^m$ and the end of application of the related control sequence $u_j(\cdot)$ at $\tau_{j+1}^c + T_{j+1}^c$. In view of Assumption 1, this condition is verified for: $\tau_{\gamma(j)+1}^m - \tau_{\gamma(j)}^m = \tau_m$, $T_{\gamma(j)}^m = T_m$, $\tau_j^c = \tau_{\gamma(j)}^m + T_{\gamma(j)}^m - \epsilon$, $\epsilon \approx 0$, $\tau_{j+1}^c - \tau_j^c = \tau_c$ and $T_{j+1}^c = T_c$. Therefore, the prediction horizon T_0^p is chosen as

$$T_0^p \geq T_c + T_m + \tau_m + \tau_c. \quad (13)$$

It is worth noting that the temporal length T_0^c of the control sequence sent in a packet can be shorter than T_0^p , as it does not need to account for the measurement MAD and MATI. With the aim of guaranteeing that a

valid control signal is always available to the embedded controller during any interval $[\tau_j^c + T_j^c, \tau_{j+1}^c + T_{j+1}^c]$, we consider the worst case interval achieved for: $T_j^c \approx 0$, $T_{j+1}^c = T_c$ and $\tau_{j+1}^c - \tau_j^c = \tau_c$. Therefore, the required control horizon T_0^c has to satisfy

$$T_0^c \geq T_c + \tau_c. \quad (14)$$

We thus define an infinite number of feedforward control signals as $\hat{u}_j(t) = \kappa(\hat{x}_{\gamma(j)}(t))$, $\forall t \in [\tau_j^c, \tau_j^c + T_0^c]$, $\forall j \in \mathbb{N}$. At each reception of a new control packet (i.e. at instants $\tau_j^c + T_j^c$), the buffer of the embedded controller is updated. Consequently, the control signal applied to the plant is given by

$$\hat{u}(t) = \hat{u}_j(t), \quad \forall t \in [\tau_j^c + T_j^c, \tau_{j+1}^c + T_{j+1}^c]. \quad (15)$$

Both the feedforward signals \hat{u}_j and the control \hat{u} are depicted at the bottom of Figure 2. Line styles are consistent with those of the estimate evolutions used to build the control signals. Vertical arrows show which estimate variable $\hat{x}_{\gamma(j)}$ is chosen for the computation of the feedforward signal \hat{u}_j at time instant τ_j^c , and which control signal \hat{u}_j is used at $\tau_j^c + T_j^c$ to update the embedded controller. In the particular example of Figure 2, it can be noticed that \hat{u}_j and \hat{u}_{j+1} are computed with respect to the same estimate \hat{x}_i since $\gamma(j) = \gamma(j+1) = i$. On the other hand, \hat{x}_{i+1} is not directly used by any control since $\gamma(j+2) = i+2$.

3.2 A reduced NCS model

The model considered so far makes use of infinitely many state estimate variables \hat{x}_i and control signals \hat{u}_j . They can be reduced to a finite number by noticing that they are all defined over compact time intervals and that “old” variables are no longer used after a while. State estimates variables are stored in a finite memory and new values are cyclically written on dismissed variables. We must prevent that a variable is accidentally reset while still in use for the computation of a control signal. In particular, $\hat{x}_{\gamma(j)}$ cannot be reset during the interval $[\tau_{\gamma(j)}^m, \tau_{j+1}^c + T_{j+1}^c]$. Hence, the dimension of such a memory, in terms of number of variables, is given by the maximum number of measurements that can be received during the life horizon T_0^p of an estimate variable. Recalling that T_0^p accounts also for the interval during which no measurements are received, whose length is bounded by τ_m , the dimension N of the memory is given by

$$N \triangleq \left\lceil \frac{T_0^p - \tau_m}{\epsilon_m} \right\rceil + 1. \quad (16)$$

Therefore, we use only N state variables x_{c_r} , $r \in \{1, \dots, N\}$, to store the state estimates. They are cyclically updated according to the following relation

$x_{c_r}(t) \triangleq \hat{x}_i(t)$ iff $t \in (\tau_i^m, \tau_{i+1}^m]$ and $\mu(i) = r$, where $\mu : \mathbb{N} \rightarrow \{1, \dots, N\}$ is defined as

$$\mu(i) \triangleq ((i-1) \bmod N) + 1. \quad (17)$$

We stress here that the theoretical model presented in this section differs from the real implementation of our control strategy. A real implementation does not require the memory to store N entire system's evolutions. It is enough to store only measurements and, in particular, at most $\left\lfloor \frac{\tau_c + T_m}{\varepsilon_m} \right\rfloor + 1 \leq N$ of them. Such a value represents the maximum number of measurements that can be received during any interval $[\tau_{\gamma(j)}^m, \tau_{j+1}^c]$. In the time instant τ_{j+1}^c the measurements received during $[\tau_{\gamma(j)}^m, \tau_{j+1}^c]$ are then used to build the prediction and the control sequence to be applied in the interval $[\tau_{j+1}^c + T_{j+1}^c, \tau_{j+2}^c + T_{j+2}^c]$. By means of the vectors $\bar{x}, x_c, e \in \mathbb{R}^{Nn}$ defined as $\bar{x} \triangleq [x^T, \dots, x^T]^T$, $x_c \triangleq [x_{c_1}^T, \dots, x_{c_N}^T]^T$ and $e = [e_1^T, \dots, e_N^T]^T \triangleq x_c - \bar{x}$, the closed-loop dynamics of the NCS can be compactly written as

$$\dot{x} = F(t, \bar{x}, e) \quad (18a)$$

$$\dot{e} = G(t, \bar{x}, e) \quad (18b)$$

$$e(\tau_i^{m+}) = H(i, e(\tau_i^m)), \quad (18c)$$

where³

$$F(t, \bar{x}, e) = f(x, u(t, e + \bar{x})) \quad (19a)$$

$$G(t, \bar{x}, e) = \begin{bmatrix} \hat{f}(e_1 + x, \kappa(e_1 + x)) - f(x, u(t, e + \bar{x})) \\ \vdots \\ \hat{f}(e_N + x, \kappa(e_N + x)) - f(x, u(t, e + \bar{x})) \end{bmatrix} \quad (19b)$$

$$H(i, e) = \begin{bmatrix} e_1 + (h(i, e_N) - e_1) \eta(i, 1) \\ e_2 + (h(i, e_1) - e_2) \eta(i, 2) \\ \vdots \\ e_N + (h(i, e_{N-1}) - e_N) \eta(i, N) \end{bmatrix}, \quad (19c)$$

where $\eta : \mathbb{N} \times \{1, \dots, N\} \rightarrow \{0, 1\}$ identifies the index of the relevant state estimate (recall (17))

$$\eta(i, r) \triangleq \begin{cases} 1 & \text{if } \mu(i) = r \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

³ In the sequel we denote by $u : \mathbb{R} \times \mathbb{R}^{Nn} \rightarrow \mathbb{R}^m$ the signal defined in (21).

The control signal u in (19a) and (19b) is given by

$$u(t, x_c) \triangleq \sum_{k=1}^N \kappa(x_{c_k}) \nu(t, k), \quad (21)$$

where $\nu : \mathbb{R}_{\geq 0} \times \{1, \dots, N\} \rightarrow \{0, 1\}$ is defined as

$$\nu(t, k) \triangleq \begin{cases} 1 & \text{if } \exists j \in \mathbb{N} \text{ s.t. } t \in (\tau_j^c + T_j^c, \tau_{j+1}^c + T_{j+1}^c] \\ & \text{and } \mu(\gamma(j)) = k \\ 0 & \text{otherwise.} \end{cases}$$

This compact notation has the advantage of involving a finite number of state variables and of fitting the framework of [21]. Note that the control signal in (15) now reads $\hat{u}(t) = u(t, x_c(t))$.

4 Main results

We start by proving that the obtained protocol (18c) and (19c) inherits the UGES property from the original one (1). All proofs are deferred to Section 6.

Proposition 1 *Under Assumption 2, the protocol modeled by the discrete-time system (18c) and (19c) is UGES and admits an associated Lyapunov function $W : \mathbb{N} \times \mathbb{R}^{Nn} \rightarrow \mathbb{R}_{\geq 0}$ given by*

$$W(k, e) \triangleq \sum_{r=1}^N W_0(k, e_r) \eta(k, r),$$

where η is defined in (20), and satisfying for all $k \in \mathbb{N}$ and all $e \in \mathbb{R}^{Nn}$:

$$a_L |e| \leq W(k, e) \leq a_H |e| \quad (22)$$

$$W(k+1, H(k, e)) \leq \rho_0 W(k, e) \quad (23)$$

$$\left| \frac{\partial W}{\partial e}(k, e) \right| \leq c, \quad (24)$$

with $a_L \triangleq \underline{a}$ for $N = 1$ and $a_L \triangleq \frac{\underline{a}}{N} \min \left\{ 1, \left(\frac{\underline{a}}{\bar{a}} \right)^2 \frac{1}{\rho_0} \right\}$ for $N > 1$, and $a_H \triangleq \bar{a}$.

Let us now present a local result on the exponential stability of the NCS (18). It provides an explicit bound (cf. (25) below) on the measurement MATI τ_m in terms of the characteristic parameters of the network-free closed-loop system, the protocol, the regularity assumptions on the dynamics and the model precision.

Theorem 1 *Assume that Assumptions 1-3 hold. Given some $R > 0$, fix $R_x = R$ and $R_u = \lambda_\kappa R$ and suppose that Assumptions 4-5 hold with these constants. Let \underline{a} , \bar{a} , ρ_0 , c , $\underline{\alpha}$, $\bar{\alpha}$, α , d , $\lambda_{f\hat{f}}$, λ_f , λ_κ , a_L , a_H be generated*

by these assumptions and by Proposition 1. Assume that the following conditions on $\tau_m, \tau_c, T_m, T_c, \varepsilon_m$ hold

$$\tau_m \in [\varepsilon_m, \tau_m^*], \quad \tau_m^* \triangleq \frac{1}{L} \ln \left(\frac{M\gamma_2 + a_L L}{M\gamma_2 + a_L \rho_0 L} \right) \quad (25)$$

$$N = \left\lceil \frac{T_c + T_m + \tau_c}{\varepsilon_m} \right\rceil + 1 \quad (26)$$

where

$$L \triangleq \frac{c}{a_L} \left(\sqrt{N} \lambda_{ff} (1 + \lambda_\kappa) + \sqrt{N} \lambda_f \right. \\ \left. + (\sqrt{N} - 1 + N - 1) \lambda_f \lambda_\kappa \right) \quad (27)$$

$$M \triangleq cN \lambda_{ff} (1 + \lambda_\kappa) \quad (28)$$

$$\gamma_2 \triangleq \frac{d}{\alpha} \sqrt{\frac{\alpha}{\alpha}} \lambda_f \lambda_\kappa. \quad (29)$$

Then, the origin of the NCS (18) is exponentially stable with radius of attraction

$$\tilde{R} \triangleq \frac{R}{K}, \quad (30)$$

where

$$K \triangleq \frac{\sqrt{2}}{1 - \gamma_1 \gamma_2} \max \{k_2 (1 + \gamma_1), k_1 (1 + \gamma_2)\} \quad (31)$$

$$\gamma_1 \triangleq \frac{\exp(L\tau_m) - 1}{a_L L (1 - \rho_0 \exp(L\tau_m))} M \quad (32)$$

$$k_1 \triangleq \frac{a_H}{\rho_0 a_L} \quad (33)$$

$$k_2 \triangleq \sqrt{\frac{\alpha}{\alpha}}. \quad (34)$$

It is important to remark that the bound (25) on the measurement MATI is also related to the dimension of the memory N , whose definition (26), obtained by (16) for $T_0^p = T_c + T_m + \tau_m + \tau_c$, embeds the other relevant communication parameters: MADs and control MATI. The pair (25)-(26) thus imposes a trade-off between the two MATIs and the MADs. The packet-based strategy aims at enlarging the control MATI τ_c , but a larger τ_c could require a larger memory N and hence could produce a lower measurement MATI τ_m . Moreover, conditions (25)-(26) bind the four relevant parameters (i.e. T_c, T_m, τ_c and τ_m) together and with the constant ε_m , namely the mTI. In particular, they require that the communication MATI τ_m is not smaller than the mTI ε_m . Furthermore, depending on the parameter R for which Assumptions 4 and 5 hold, an explicit estimate \tilde{R} of the radius of attraction can be computed, cf. (30). Note that, since Theorem 1 guarantees only local properties, Assumption 3 could be relaxed to *local* exponential stability of the nominal plant, over a sufficiently large

domain of attraction.

The following proposition establishes that the MATI and memory requirements of the previous theorem can always be satisfied.

Proposition 2 *Given any $R > 0$, the parameters $\tau_m, \tau_c, T_m, T_c, \varepsilon_m$ can always be picked small enough to satisfy conditions (25)-(26).*

In general, the radius of attraction \tilde{R} of the resulting NCS guaranteed by Theorem 1 cannot be arbitrarily specified due to the possible dependency of the constants L and M (and consequently K) in the parameter R ruling the domain on which Assumptions 4 and 5 hold. To see this more clearly, consider, for instance, the case of K proportional to R . Relation (30) shows that, in this case, the radius of attraction \tilde{R} would be a constant irrespective of the size of R . One could even imagine that, in some situations, \tilde{R} actually shrinks when R is enlarged. Hence, in order to ensure that the set of initial conditions can be arbitrarily enlarged, we must add some constraints on the growth rate of the constant K or, equivalently, on some of the Lipschitz constants. After reporting a definition of semiglobal exponential stability which is adapted to our NCS framework, we present our main result in this regard in Theorem 2.

Definition 1 *The NCS (18) is said to be semiglobally exponentially stable if, for any $\tilde{R} > 0$, there exist positive constants $\tau_m^*(\tilde{R}), \tau_c^*(\tilde{R}), T_m^*(\tilde{R}), T_c^*(\tilde{R})$ and $\varepsilon_m^*(\tilde{R})$, as introduced in Assumption 1, such that its origin is exponentially stable on $B_{\tilde{R}}$.*

Theorem 2 *Suppose that Assumptions 1-4 hold for all $R_x, R_u > 0$ and that there exists $\sigma \in [0, 1)$ such that*

$$\lim_{s \rightarrow \infty} \frac{\lambda_f(s) \lambda_\kappa(s)}{s^\sigma} < \infty. \quad (35)$$

Then, the NCS (18) is semiglobally exponentially stable.

The above result guarantees that, provided sufficient regularity of the dynamics (i.e. Lipschitz constants sub-linear in the size of the domain over which they are computed), any prescribed compact domain of attraction can be reached if MADs and MATIs are small enough.

4.1 Robustness and Sampling

From classical robustness results of exponentially stable hybrid systems [9], it follows that, given any $\tilde{R} > 0$, there exists a continuous function $\psi : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$, satisfying $\psi(s) > 0$ for all $\psi \in (0, \tilde{R})$, such that (18) remains exponentially stable on $B_{\tilde{R}}$ even in presence of measurement errors⁴ d_m and actuation errors d_a , as

⁴ Due to space constraint, the resulting system is not given explicitly here, but can be derived directly from the consid-

long as $\max\{|d_m(t, X)|, |d_a(t, X)|\} \leq \psi(|X|)$, for all $t \geq 0$, where $X \triangleq (\bar{x}^T, e^T)^T$. In other words there exist $k_1, k_2 > 0$ such that, given any $X_0 \in B_{\tilde{R}}$, the solutions of the perturbed system satisfy $|X(t)| \leq k_1|X_0|e^{-k_2t}$ for all $t \geq 0$. In particular, if $X_0 \in B_{\tilde{R}/2k_1}$, it holds that $X(t) \in B_{\tilde{R}/2}$ for all $t \geq 0$. Now, given any $\epsilon > 0$, let $\underline{\psi} \triangleq \min_{s \in [\epsilon, \tilde{R}/2]} \psi(s)$. Then $\underline{\psi} > 0$ and it holds that, if $|d_m|, |d_a| \leq \underline{\psi}$, then $|X(\tau)| \in [\epsilon, \tilde{R}/2] \forall \tau \in [0, t] \Rightarrow |X(t)| \leq k_1|X_0|e^{-k_2t}$. Consequently, for all $X_0 \in B_{\tilde{R}/2k_1}$, the solutions of the perturbed system satisfy $|X(t)| \leq k_1|X_0|e^{-k_2t} + \epsilon$ provided that $|d_m|, |d_a| \leq \underline{\psi}$. This establishes some robustness with respect to sufficiently small measurement errors and allows to address explicitly the problem of sampling required to encode the control prediction into a packet for its transmission over the network. Indeed, consider the continuous-time control u in (21) and its sampled version, at constant sampling period $\Delta > 0$, u^Δ . Recalling that, from the definition of \bar{x} and e (see before (18)), $X_0 \in B_{\tilde{R}/2k_1} \Rightarrow X(t) \in B_{\tilde{R}/2} \Rightarrow x_c(t) \in B_{\tilde{R}}$, for all $t \geq 0$, we get from (21) and the local Lipschitz of κ that $|u(t) - u^\Delta(t)| \leq \lambda(\Delta)$ where λ denotes a continuous function satisfying $\lambda(0) = 0$. In particular, the actuation error resulting from the sampling of the control packet can be made smaller than ψ by picking a sufficiently small sampling period Δ , thus yielding $|X(t)| \leq k_1|X_0|e^{-k_2t} + \epsilon$. Since both \tilde{R} and ϵ are arbitrary in the above reasoning, semi-global practical exponential stability follows. In other words, a sufficiently fast sampling of the control sequence (hence a sufficiently large packet size) allows to reach any arbitrary precision from any prescribed compact set of initial conditions. Moreover, recent results [13], [22] suggest that a sufficiently fast sampling may actually yield asymptotic stability. Nonetheless, the modeling involved in the approach presented here (in particular the resetting of the control input at each arrival of a new packet) does not allow a direct application of the framework of those papers. A more precise estimate of the sampling frequency vs. the size of the region of attraction and of the steady-state error is definitely an important subject left for future research.

5 Case Study

The exploitation of the packet payload and the model-based predictive strategy presented in this paper can improve the MATI bounds obtained by sending a single control value in each packet. Let us illustrate such improvement by comparing our bounds with those computed in [29] for a Ch-47 Tandem-Rotor Helicopter. The linearized model describing the helicopter can be written as $\dot{x} = Ax + Bu$ with

$$A = \begin{bmatrix} -0.02 & 0.005 & 2.4 & -32 \\ -0.14 & 0.44 & -1.3 & -30 \\ 0 & 0.018 & -1.6 & 1.2 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.14 & -0.12 \\ 0.36 & -8.6 \\ 0.35 & 0.009 \\ 0 & 0 \end{bmatrix}.$$

Our formalism requires sending, in a distributed fashion, the state vector $x \in \mathbb{R}^4$, while in [29] authors use an exponentially stabilizing static controller relying only

on the output $\mathbb{R}^2 \ni y = Cx$ with $C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 57.3 & 0 \end{bmatrix}$. To

express this very controller in terms of x we write $u = KCx$, with $K = \begin{bmatrix} -12.7177 & -45.0824 \\ 63.5123 & 25.9144 \end{bmatrix}$ given in [29].

We assume the state x is partitioned as $x = [x_1^T, x_2^T]^T$ with $x_i \in \mathbb{R}^2$ and it is transmitted by means of two links ($\ell = 2$) ruled by the RR protocol. We also assume, without lack of generality, that x_1 is sent through the link 1 and x_2 through the link 2. The bound on the MATI provided in [21] is $\tau_{[21]}^* = 1.20 \cdot 10^{-5}$ s and the improvement in [29] provides $\tau_{[29]}^* = 2.81 \cdot 10^{-4}$ s. In the ideal case of a linear system with equidistant transmissions and RR protocol, the *constant* MATI can be computed following the argument in [21, Section VII-A]. For the present case study we get $\tau_{\text{single}}^* \simeq 1.13 \cdot 10^{-3}$ s.

In order to appreciate the improvement induced by the packet payload exploitation, let us compute the MATI bound according to expression (25). For the comparison to be fair, we assume zero delays ($T_m = T_c = 0$). We also fix $\varepsilon_m \simeq \tau_m^*$ and τ_c slightly less than ε_m (just to keep $N = 1$ in (26)). The relevant protocol parameters, according to Assumption 2, are $\underline{a} = 1$, $\bar{a} = \sqrt{\ell}$, $\rho_0 = \sqrt{(\ell - 1)/\ell}$, $c = \sqrt{\ell}$ (see [21] and recall they depend only on the number of links not on the dimension of the vectors sent through them). As concerns the nominal stability parameters in Assumption 3, we have $\underline{\alpha} = 9 \cdot 10^{-4}$, $\bar{\alpha} = 399.22$, $\alpha = 1$, $d = 2\bar{\alpha}$ (see [29]). The Lipschitz constants required by Assumption 4 are $\lambda_f = \max\{|A|, |B|\} = 43.8904$ and $\lambda_\kappa = |KC| \simeq 2980$. We assume also to have a perfect model: $\hat{f} = f$ and $\lambda_{f\hat{f}} = 0$ (see Assumption 5). With $N = 1$, the constants in (27) and (28) become $L = \frac{c}{a_L} (\lambda_{f\hat{f}}(1 + \lambda_\kappa) + \lambda_f) = \frac{c}{\underline{a}} \lambda_f$ and $M = c\lambda_{f\hat{f}}(1 + \lambda_\kappa) = 0$. Thus, the MATI bound (25) is given by $\tau_m^* = \frac{a}{c\lambda_f} \ln\left(\frac{1}{\rho_0}\right) \simeq 5.58 \cdot 10^{-3}$ s, i.e. about 20 times larger than $\tau_{[29]}^*$ - or, in other terms, our method would require sending ca. 20 times less packets to stabilize the system. In reality, the exploitation of the packet payload often induces a more substantial improvement than that shown by the bounds above. If we compare the constant MATI⁵ achievable with our predictive control strategy with a classical single control

eration of measurement and actuation errors in (18). In the sequel, this system is referred to as the perturbed system.

⁵ Even in this case we used the argument in [21, Section VII-A].

value technique, we have $\tau_{\text{mult}}^* \simeq 1.3105$ s, hence a theoretical improvement of $\frac{\tau_{\text{mult}}^*}{\tau_{\text{single}}^*} \simeq 1160$.

6 Proofs

6.1 Proof of Proposition 1

For $N = 1$, $W(k, e) = W_0(k, e)$ and the thesis follows from Assumption 2. For $N > 1$, let us consider any $s \in \{1, \dots, N-1\}$ satisfying $\eta(k, s) = 1$ (with η defined in (20)) for some $k \in \mathbb{N}_{\geq 1}$. Then $W(k, e) = W_0(k, e_s)$ and for $k+1$ we have $\eta(k+1, s+1) = 1$ and $W(k+1, H(k, e)) = W_0(k+1, h(k, e_s)) \leq \rho_0 W_0(k, e_s) = \rho_0 W(k, e_s)$, which establishes (23). The inequality $W(k, e) = W_0(k, e_s) \leq \bar{a}|e_s| \leq \bar{a}|e|$ is easily verified (thus $a_H = \bar{a}$), while the left inequality in (22) requires to consider the evolution of the system. Recall that at step k the $(s-1)$ -th variable is updated, hence the s -th variable is left unchanged for $N-1$ intervals, the $(s+1)$ -th variable for $N-2$ intervals, etc. Summarizing, the following relations hold:

$$\begin{aligned} e_{s-1}(k) &= h(k-1, e_{s-2}(k-1)) \\ e_{s-2}(k) &= e_{s-2}(k-1) = h(k-2, e_{s-3}(k-2)) \\ &\vdots \\ e_1(k) &= e_1(k-1) = \dots = e_1(k-s+2) \\ &= h(k-s+1, e_N(k-s+1)) \\ e_N(k) &= e_N(k-1) = \dots = e_N(k-s+1) \\ &= h(k-s, e_{N-1}(k-s)) \\ &\vdots \\ e_{s+1}(k) &= e_{s+1}(k-1) = \dots = e_{s+1}(k-N+2) \\ &= h(k-N+1, e_s(k-N+1)) \\ e_s(k) &= e_s(k-1) = \dots = e_s(k-N+1) \\ &= h(k-N, e_{s-1}(k-N)). \end{aligned}$$

This permanency allows us to write $N-1$ of the previous relations in terms only of the variables computed at instant k , and hence to suppress the dependency from k : $e_{s-1} = h(k-1, e_{s-2})$, \dots , $e_{s+1} = h(k-N+1, e_s)$. By the previous relations and inequality (3) we have $W_0(k, e_{s-1}) = W_0(k, h(k-1, e_{s-2})) \leq \rho_0 W_0(k-1, e_{s-2})$, and consequently $W_0(k-1, e_{s-2}) \geq \frac{W_0(k, e_{s-1})}{\rho_0}$. Analogously we can write

$$W_0(k-N+1, e_s) \geq \frac{W_0(k-N+2, e_{s+1})}{\rho_0}. \quad (36)$$

Inequality (36) is given in terms of W_0 computed in different time instants ($k-N+1$ and $k-N+2$). In order for this relation to refer to the same time k , let us recall that for all k we can write $\underline{a}|e| \leq W_0(k, e) \leq \bar{a}|e|$ and

hence for all e, q and k

$$W_0(q, e) \geq \frac{a}{\bar{a}} W_0(k, e). \quad (37)$$

Whereby, using (36) and (37) we have $W_0(k, e_s) \geq \frac{a}{\bar{a}} W_0(k-N+1, e_s) \geq \frac{a}{\bar{a}\rho_0} W_0(k-N+2, e_{s+1}) \geq \left(\frac{a}{\bar{a}}\right)^2 \frac{1}{\rho_0} W_0(k, e_{s+1})$. Iterating the application of (36) and applying two times (37), we produce the following chain of inequalities $W_0(k, e_s) \geq \left(\frac{a}{\bar{a}}\right)^2 \frac{1}{\rho_0} W_0(k, e_{s+1}) \geq \left(\frac{a}{\bar{a}}\right)^2 \frac{W_0(k, e_{s+2})}{\rho_0^2} \geq \dots \geq \left(\frac{a}{\bar{a}}\right)^2 \frac{1}{\rho_0^{N-1}} W_0(k, e_{s-1})$. Using the previous relations we can write the sought inequality and compute a_L :

$$\begin{aligned} W(k, e) &= W_0(k, e_s) = \frac{1}{N} (NW_0(k, e_s)) \\ &\geq \frac{1}{N} \left(W_0(k, e_s) + \left(\frac{a}{\bar{a}}\right)^2 \frac{1}{\rho_0} W_0(k, e_{s+1}) + \dots \right. \\ &\quad \left. + \left(\frac{a}{\bar{a}}\right)^2 \frac{1}{\rho_0^{N-1}} W_0(k, e_{s-1}) \right) \\ &\geq \frac{a}{N} \left(|e_s| + \left(\frac{a}{\bar{a}}\right)^2 \frac{1}{\rho_0} |e_{s+1}| + \dots + \left(\frac{a}{\bar{a}}\right)^2 \frac{1}{\rho_0^{N-1}} |e_{s-1}| \right) \\ &\geq \frac{a}{N} \min \left\{ 1, \left(\frac{a}{\bar{a}}\right)^2 \frac{1}{\rho_0}, \dots, \left(\frac{a}{\bar{a}}\right)^2 \frac{1}{\rho_0^{N-1}} \right\} |e| \\ &\geq \frac{a}{N} \min \left\{ 1, \left(\frac{a}{\bar{a}}\right)^2 \frac{1}{\rho_0} \right\} |e|, \end{aligned}$$

where we used the fact that $\frac{a}{\bar{a}}, \rho_0 < 1$. This establishes the left inequality in (22). Finally we write

$$\begin{aligned} \left| \frac{\partial W}{\partial e}(k, e) \right| &= \left| \frac{\partial W_0}{\partial e}(k, e_s) \right| \\ &= \left| \left[0, \dots, 0, \frac{\partial W_0}{\partial e_s}(k, e_s), 0, \dots, 0 \right]^T \right| \leq c, \end{aligned}$$

which establishes (24) and concludes the proof.

6.2 Proof of Theorem 1

The proof consists of the following 4 steps:

1. Show that the system (18b)-(18c) is locally input-to-state exponentially stable with linear gain from x to e , provided that solutions remain inside B_R ;
2. Show that the system (18a) is locally input-to-state exponentially stable with linear gain from e to x , provided that solutions remain inside B_R ;
3. Show by means of small gain arguments that the overall system (18) is locally exponentially stable inside B_R ;
4. Show that solutions remain indeed inside B_R as long as initial conditions are in $B_{\tilde{R}}$.

Step 1 Let us consider any $r \in \{1, \dots, N\}$ such that $\nu(t, r) = 1$ for some $t \geq 0$. In view of (21) we can simply write $u(t, e + \bar{x}) = \kappa(e_r + x)$. Since $|e_s| \leq |e|$ for all $s \in \{1, \dots, N\}$, in light of (19b) and of Assumptions 4 and 5, we have for all $|(x^T, e^T)| \leq R$

$$\begin{aligned}
& \frac{\partial W}{\partial e} G(t, \bar{x}, e) \leq \left| \frac{\partial W}{\partial e} \right| |G(t, \bar{x}, e)| \\
& \leq c \sum_{s=1}^N \left| \hat{f}(e_s + x, \kappa(e_s + x)) - f(x, \kappa(e_r + x)) \right| \\
& \leq c \sum_{s=1}^N \left| \hat{f}(e_s + x, \kappa(e_s + x)) - f(e_s + x, \kappa(e_s + x)) \right| \\
& + c \sum_{s=1}^N |f(e_s + x, \kappa(e_s + x)) - f(x, \kappa(e_r + x))| \\
& \leq c \sum_{s=1}^N \lambda_{f\hat{f}} (|e_s| + |x| + \lambda_\kappa (|e_s| + |x|)) \\
& + c \sum_{s=1}^N \lambda_f (|e_s| + \lambda_\kappa |e_s - e_r|) \\
& \leq cN\lambda_{f\hat{f}} (1 + \lambda_\kappa) |x| + c \left(\sqrt{N}\lambda_{f\hat{f}}(1 + \lambda_\kappa) + \sqrt{N}\lambda_f \right. \\
& \left. + \left(\sqrt{N-1} + N-1 \right) \lambda_f \lambda_\kappa \right) |e| \\
& \leq |\tilde{y}| + LW(i, e),
\end{aligned}$$

where L is given by (27) and $\tilde{y} \triangleq Mx$ with M given by (28). In the light of footnote 8 in [21] we have that, for all $i \in \mathbb{N}$ and almost all t , $\frac{d}{dt} W(i, e(t)) \leq LW(i, e(t)) + |\tilde{y}(t)|$. From Proposition 6 and the proof of Proposition 7 in [21], we conclude the input-output stability of system (18b) from \tilde{y} to W with $\exp\text{-}\mathcal{KL}$ function and linear gain, provided that $|(x^T(t), e^T(t))| \leq R$ at all time. More precisely, for any $t \in [\tau_i^m, \tau_{i+1}^m]$ and $i \geq k \geq 0$ arbitrarily chosen, we have that $t - \tau_k^m \leq (i - k + 1)\tau_m$ and, as long as $|(x^T(t), e^T(t))| \leq R$,

$$\begin{aligned}
W(i, e(t)) & \leq \exp(L\tau_m)\lambda_{(k+1) \rightarrow i} W(k, e(\tau_k^m)) \\
& + \frac{\exp(L\tau_m) - 1}{L(1 - \rho_0 \exp(L\tau_m))} \|\tilde{y}[\tau_k^m, t]\|_{\mathcal{L}_\infty},
\end{aligned}$$

with

$$\begin{aligned}
\lambda_{(k+1) \rightarrow i} & \triangleq (\rho_0 \exp(L\tau_m))^{i-k} \\
& \leq \frac{\exp\left(\ln(\rho_0 \exp(L\tau_m)) \frac{t - \tau_k^m}{\tau_m}\right)}{\rho_0 \exp(L\tau_m)}.
\end{aligned}$$

Note that (25) ensures in particular that $\rho_0 \exp(L\tau_m) < 1$. The previous expression holds in particular for $k = 0$. Recalling the definition of \tilde{y} and the inequalities (22), we can write, as long as $|(x^T(t), e^T(t))| \leq R$,

$$|e(t)| \leq k_1 \exp(-\lambda_1(t - \tau_0^m)) |e_0| + \gamma_1 \|x[\tau_0^m, t]\|_{\mathcal{L}_\infty}, \quad (38)$$

with γ_1 and k_1 given by (32) and (33) respectively, and $\lambda_1 \triangleq -\frac{\ln(\rho_0 \exp(L\tau_m))}{\tau_m} > 0$.

Step 2 Let us consider the Lyapunov function V of Assumption 3. In view of Assumption 4, the total derivative of V along the solutions of (18a) yields for all $|(x^T, e^T)| \leq R$

$$\begin{aligned}
& \frac{\partial V}{\partial x} F(t, \bar{x}, e) = \frac{\partial V}{\partial x} f(x, \kappa(e_r + x)) \\
& = \frac{\partial V}{\partial x} f(x, \kappa(x)) + \frac{\partial V}{\partial x} [f(x, \kappa(e_r + x)) - f(x, \kappa(x))] \\
& \leq -\alpha |x|^2 + d\lambda_f \lambda_\kappa |x| |e_r| \\
& \leq -\alpha |x|^2 + d\lambda_f \lambda_\kappa |x| |e_r| + \left(\sqrt{\frac{\alpha}{2}} |x| - \frac{d\lambda_f \lambda_\kappa}{\sqrt{2\alpha}} |e_r| \right)^2 \\
& = -\frac{\alpha}{2} |x|^2 + \frac{d^2 \lambda_f^2 \lambda_\kappa^2}{2\alpha} |e_r|^2 \\
& \leq -\frac{\alpha}{2\alpha} V(x) + \frac{d^2 \lambda_f^2 \lambda_\kappa^2}{2\alpha} |e|^2.
\end{aligned} \quad (39)$$

Whereby, applying the comparison lemma, we get, as long as $|(x^T(t), e^T(t))| \leq R$,

$$\begin{aligned}
V(x(t)) & \leq \exp\left(-\frac{\alpha}{2\alpha}(t - \tau_0^m)\right) V(x(\tau_0^m)) \\
& + \frac{d^2 \lambda_f^2 \lambda_\kappa^2}{2\alpha} \int_{\tau_0^m}^t \exp\left(-\frac{\alpha}{2\alpha}(t - s)\right) |e(s)|^2 ds \\
& \leq \exp\left(-\frac{\alpha}{2\alpha}(t - \tau_0^m)\right) V(x_0) \\
& + \frac{d^2 \lambda_f^2 \lambda_\kappa^2}{\alpha^2} \left(1 - \exp\left(-\frac{\alpha}{2\alpha}(t - \tau_0^m)\right)\right) \|e[\tau_0^m, t]\|_{\mathcal{L}_\infty}.
\end{aligned}$$

Hence, recalling Assumption 3, we can write, as long as $|(x^T(t), e^T(t))| \leq R$,

$$|x(t)| \leq k_2 \exp(-\lambda_2(t - \tau_0^m)) |x_0| + \gamma_2 \|e[\tau_0^m, t]\|_{\mathcal{L}_\infty}, \quad (40)$$

with γ_2 and k_2 given by (29) and (34) respectively, and $\lambda_2 \triangleq \frac{\alpha}{4\alpha} > 0$.

Step 3 By means of a local version of Corollary 1 in [21], we can conclude that the NCS system (18) is locally exponentially stable if the small gain condition $\gamma_1 \gamma_2 < 1$ is verified and $|(x^T(t), e^T(t))| \leq R$ for all t . In view of (29) and (32), it is easy to see that the previous inequality is satisfied for every $\tau_m \in [\varepsilon_m, \tau_m^*]$ compatible with conditions (25)-(26). The value of τ_m^* in (25) can be found by solving in τ_m the small gain condition.

Step 4 Finally, we can compute the set of initial conditions for which trajectories indeed remain inside B_R . Recalling the inequalities (38) and (40) we can write

$$\begin{aligned}
\|x[\tau_0^m, t]\|_{\mathcal{L}_\infty} & \leq k_2 |x_0| + \gamma_2 \|e[\tau_0^m, t]\|_{\mathcal{L}_\infty} \\
\|e[\tau_0^m, t]\|_{\mathcal{L}_\infty} & \leq k_1 |e_0| + \gamma_1 \|x[\tau_0^m, t]\|_{\mathcal{L}_\infty},
\end{aligned}$$

and

$$\left\| \begin{array}{l} x[\tau_0^m, t] \\ e[\tau_0^m, t] \end{array} \right\|_{\mathcal{L}_\infty} \leq \|x[\tau_0^m, t]\|_{\mathcal{L}_\infty} + \|e[\tau_0^m, t]\|_{\mathcal{L}_\infty} \leq K \begin{array}{l} x_0 \\ e_0 \end{array}$$

with K given by (31). Consequently, in order to ensure that the evolution of the system does not exit the ball $B_{\tilde{R}}$, it is sufficient to impose that $K \|(x_0^T, e_0^T)\| < \tilde{R}$, or equivalently that $(x_0^T, e_0^T) \in B_{\tilde{R}} \subseteq B_R$ with $\tilde{R} = \frac{R}{K}$ (cf. (30)).

6.3 Proof of Proposition 2

Let us name τ_m^1 the value assumed by τ_m^* (see (25)) for $N = 1$. Conditions (25)-(26) are satisfied for every $\tau_m, T_c, T_m, \tau_c, \varepsilon_m$ such that $\tau_m \in [\varepsilon_m, \tau_m^1)$ and $0 < T_c + T_m + \tau_c < \varepsilon_m$, which is always feasible for sufficiently small values of these parameters.

6.4 Proof of Theorem 2

This proof strongly relies on that of Theorem 1. According to Definition 1, we must show that for any arbitrarily fixed set of initial conditions we can find suitable values for the parameters $\tau_m, T_c, T_m, \tau_c, \varepsilon_m$ ensuring the exponential stability of the NCS on the chosen set. Let us consider $R > 0$ as a free variable. By the small gain condition $\gamma_1(R)\gamma_2(R) < 1$ (cf. Step 3 of the proof of Theorem 1) and recalling expression (32) of $\gamma_1(R)$, we have that, for any $\delta \in (0, 1)$ independent of R , we can find a constant

$$\tau_m^*(R, \delta) \triangleq \frac{1}{L(R)} \ln \left(\frac{M(R)\gamma_2(R) + (1 - \delta)a_L L(R)}{M(R)\gamma_2(R) + (1 - \delta)a_L \rho_0 L(R)} \right),$$

such that for every $\tau_m \in (0, \tau_m^*(R, \delta)]$ $\gamma_1(R)\gamma_2(R) \leq 1 - \delta$. In a way similar to Proposition 2 we can show that it is always possible, for any fixed R , to find a set of parameters $\tau_m, T_c, T_m, \tau_c, \varepsilon_m$ satisfying the previous condition and the conditions (25)-(26).

All the Lipschitz constants of Assumptions 4 and 5 are non-decreasing functions of R . According to the inequality (35), we distinguish two cases: $\sigma = 0$ and $\sigma \in (0, 1)$.

Case $\sigma = 0$. From (35) and (29) we have $\lim_{R \rightarrow \infty} \gamma_2(R) \triangleq \bar{\gamma}_2 < \infty$, then $\tau_m^*(R, \delta) \geq \tau_{m\bar{\gamma}_2}^*(R, \delta)$ with

$$\tau_{m\bar{\gamma}_2}^*(R, \delta) \triangleq \frac{1}{L(R)} \ln \left(\frac{M(R)\bar{\gamma}_2 + (1 - \delta)a_L L(R)}{M(R)\bar{\gamma}_2 + (1 - \delta)a_L \rho_0 L(R)} \right)$$

and for every $\tau_m \in (0, \tau_{m\bar{\gamma}_2}^*(R, \delta))$, $\gamma_1(R) < \frac{1 - \delta}{\bar{\gamma}_2}$. This means that $K(R)$ of (31) is bounded by the constant $\bar{K} \triangleq \frac{\sqrt{2}}{\delta} \max \left\{ k_2 \left(1 + \frac{1 - \delta}{\bar{\gamma}_2} \right), k_1 (1 + \bar{\gamma}_2) \right\}$, thus allowing the radius \tilde{R} of the set of initial conditions to be

arbitrarily chosen. Indeed, once \tilde{R} is fixed, R can easily be computed as $R = \bar{K}\tilde{R}$ (cf. (30)). Such a value of R is a function of \tilde{R} , hence, it can be used to explicitly compute the parameters $\tau_m^*(\tilde{R}), T_c^*(\tilde{R}), T_m^*(\tilde{R}), \tau_c^*(\tilde{R}), \varepsilon_m^*(\tilde{R})$ required by Definition 1.

Case $\sigma \in (0, 1)$. From (35) and (29) we have $\lim_{R \rightarrow \infty} \gamma_2(R) = \infty$ and $\lim_{R \rightarrow \infty} \frac{\gamma_2(R)}{R^\sigma} \triangleq h < \infty$. We can chose $\tau_m \in (0, \tau_m^*(R, \delta))$ such that $\lim_{R \rightarrow \infty} \gamma_1(R) = 0$ (cf. (32)) to ensure that $\gamma_1(R)\gamma_2(R) \leq 1 - \delta$. $\gamma_2(R)$ being a non-decreasing function of R , there exists an \bar{R} such that for any $R > \bar{R}$, $\max\{k_2(1 + \gamma_1(R)), k_1(1 + \gamma_2(R))\} = k_1(1 + \gamma_2(R))$. Let us consider the case of $R > \bar{R}$. By the condition (30), for \tilde{R} to be arbitrary enlargeable, it must hold $\lim_{R \rightarrow \infty} \frac{R}{K(R)} = \infty$. By the previous relations we have that, for any $R > \bar{R}$ and any $\tau_m \in (0, \tau_m^*(R, \delta))$, $K(R) \leq \frac{\sqrt{2}}{\delta} k_1 (1 + \gamma_2(R))$. Hence, recalling (31),
$$\lim_{R \rightarrow \infty} \frac{R}{K(R)} \geq \lim_{R \rightarrow \infty} \frac{R}{\frac{\sqrt{2}}{\delta} k_1 (1 + \gamma_2(R))} = \lim_{R \rightarrow \infty} \frac{\delta R^{1 - \sigma}}{\sqrt{2} k_1} \left(\frac{1}{R^\sigma} + \frac{\gamma_2(R)}{R^\sigma} \right)^{-1} = \lim_{R \rightarrow \infty} R^{1 - \sigma} \frac{\delta}{h \sqrt{2} k_1} = \infty$$
 as desired. For any arbitrarily chosen $\tilde{R} > \bar{R}$ with $\tilde{R}^* \triangleq \frac{\tilde{R}}{K(\tilde{R})}$, we can compute R by solving (31). Once again, such a value of R is a function of \tilde{R} , hence, it can be used to explicitly compute the parameters $\tau_m^*(\tilde{R}), T_c^*(\tilde{R}), T_m^*(\tilde{R}), \tau_c^*(\tilde{R}), \varepsilon_m^*(\tilde{R})$ required by Definition 1. Therefore, semiglobal exponential stability is proved for some $\tilde{R} > \tilde{R}^*$. In view of the Definition 1 it is apparent that the same property also holds for all $\tilde{R} \leq \tilde{R}^*$. Indeed, it is enough to fix all the required constants to the value they assume for any $\tilde{R} > \tilde{R}^*$ (for instance for $\tilde{R} = 2\tilde{R}^*$) to have that the system is exponentially stable on $B_{\tilde{R}}$.

7 Conclusions

The problem of stabilizing nonlinear time-invariant plants over a limited-bandwidth packet-switching network has been considered. The proposed adoption of feedforward control sequences exploiting the packet-switching nature of the network allows to send larger packets less frequently. This model-based approach remotely computes a predictive control signal on a given time horizon. We considered a robustness problem, where the plant uncertainty is given a priori, and we provided a bound on the combined effects of the MATI and MAD as a function of the basin of attraction and the model precision. The great improvement to the MATI induced by our control strategy with respect to classic techniques sending one control value per packet, has been verified by means of a case study. Our future research will focus on the exploitation of the packetization of measurements (i.e. sequence of measurements in the same packet) to further reduce the bandwidth occupation and to better cope with model parameter variations.

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