Intrinsic Contact Sensing for Soft Fingers

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Abstract

In this paper the basic mathematic relationships of "intrinsic" (or "force-based") contact sensing are discussed. While conventional tactile sensing devices are designed to provide information about local phenomena caused by contact (typically concerning the spatial distribution of normal pressures), intrinsic contact sensing detects a few global quantities relating to the interactions of two bodies in contact. However, these quantities are very important from the point of view of manipulation control. The paper addresses the geometric-mathematical problem of detecting these quantities starting from force/torque measurements and from the geometric description of one of the contacting surfaces. Two methods for solving the intrinsic contact sensing problem are discussed. The first method is able to give exact results for contacts of the "hard finger" type, while it is shown to be only approximate for "soft finger" contacts: a formula for estimating the extent of such approximation error is provided. A second, novel solution method is presented, which applies to soft fingers with ellipsoidal surface, and is capable of yielding exact solutions to the problem. Finally, some implementation issues and applications of intrinsic tactile sensing to fine manipulation operations are reviewed.

1 Introduction

Contact is the fundamental physical phenomenon on which manipulation relies. Contact is not only the means by which parts of the manipulator (most often its fingers) can exert forces on objects so as to control their motion, but also a source of sensorial information about manipulated objects. Therefore, robotics researchers have been devoting much attention in the recent past to replicating the capability of humans of sensing contact through tactile sensors.

Most attempts conducted so far used the human example not only as a performance target for their devices, but also as a model to follow in designing them. Thus, conventional contact sensing devices usually consist of many simple pressure transducers, arrayed on or in proximity of the surface of the robot finger, in much the same way that Pacini, Merkel or Meissner tactile corpuscles are located in our skin. Although several such skin-like tactile sensors have been realized showing good results, there are some limitations inherent in this approach. Besides technological difficulties in building a high resolution sensor with the desired shape and material compliance, the high number (ranging typically from several tens to hundreds) of transducing elements of a skin-like sensor is an obstacle to its use in real time tasks.

Although the "tactile imaging" capability of skin-like sensors can be useful for some advanced perceptual tasks (e.g., for identifying features of an explored object which are smaller than the robot finger itself), the large majority of manipulation tasks do not require such sophisticated information. What is really needed in most cases is a knowledge of some global aspects of contact, that is the overall effects of punctual actions exerted between the bodies in the contact zone: in short, where the finger is touching, how large the contact force is, and how it is directed. It can be observed that such simplified information is pretty much the same which is felt by a human being wearing gloves or thimbles for his/her work.

Salisbury [1] first proposed the idea of collecting the synthetic data necessary to manipulation control by means of relatively simple sensing devices. He showed that starting from the measurements of a six-axis force/torque sensor, and by knowing the geometry of the finger surface, some of the desired information could be elicited. Since the force/torque sensor can be placed far from the surface where contact occurs, and usually it is *inside* the fingertip, this type of contact sensors have been designated with the term *intrinsic*, mutuated from human anatomy [2]. The sensing method proposed by [1] refers to a model of finger-object interactions where only forces are transmitted through contact (*point contact with friction*, or *hard finger* model [3]). This is not the most general case. In fact, if the finger and/or the object have compliant surface (*soft finger*), significant torques can also be exerted between the two. For a soft finger made with high-frictional material, the method proposed in [1] is not satisfactory, since errors in identifying the contact locations may result. Incidentally, it should be noted that robot fingers are often purposely built with compliant and high-friction materials, so as to allow the largest possible grasp stability.

In this paper the application of the intrinsic contact sensing approach to soft fingers is examined. In section 2 an analysis of the model of soft finger contact is carried out, aimed at clarifying the assumptions that are made and to define the quantities that completely define a contact under this model. In particular, the concept of *contact centroid* is introduced in order to render the definition of "soft finger" contact more precise. In section 3 the problem of extracting the desired information from force/torque measurements is formulated. The hard finger solution of [1] is revisited in section 4. This solution is not theoretically correct if applied to soft fingers contacts. However, the method is shown to give approximate results, and an estimate of the extent of the approximation error is derived. In section 5 a novel method is presented, which applies to soft fingers with ellipsoidal surface, and is capable of yielding exact solutions to the problem. The solving algorithms are also given for some particular and limit cases of ellipsoidal surfaces, such as spheres, cylinders and planes.

Force-based contact sensors have been actually implemented [4, 2, 5], and effectively employed in robotic hands. Although the main purpose of this paper is to provide the theoretical basis and the resolving formulas of intrinsic contact sensing for soft fingers, some implementation issues and applications of such sensors to fine manipulation operations are discussed in the final section.

2 The Soft Finger Contact Model

Consider the contact of two elastic bodies depicted in figure 1. Assume at first that the two bodies have non-conformal surfaces, so that, when brought into contact by a negligibly small force, they touch at a single point, u. Let u be taken as the origin of a cartesian reference frame u_{xyz}, whose axis u_z is chosen to coincide with the normal direction common to the two surfaces at u. The u_{xy} plane is therefore tangent to the surfaces, and is referred to as the *osculating* plane [6]. Relative motions v_x, v_y along the tangential axes are commonly called *(translational) slip*, while rotations ω_x, ω_y about the same axes are *rolling* motions. A relative rotation ω_z about the normal axis is termed *spin*, while a

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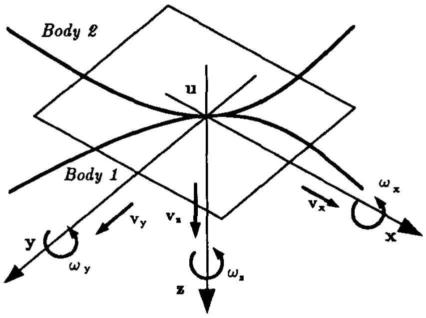

Figure 1: Initial contact between non-conformal surfaces.

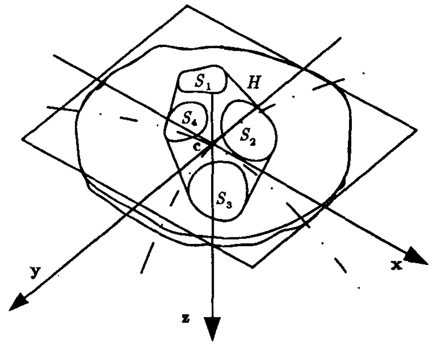

Figure 2: The contact convex hull and the centroid of contact.

motion v_z is sometimes called *approach*. As bodies are pressed by a finite intensity force, contact is made over an area S of finite extension. At each point $\mathbf{r}$ of the contact area, bodies mutually exert *tractions* $\mathbf{h} = \mathbf{h}(\mathbf{r}) = (h_x, h_y, h_z)^T$. The h_z component is usually referred to as *pressure*, while h_x and h_y are the *friction* components. The overall (resultant) contact force vector $\mathbf{p}$ is defined as:

$$\mathbf{p} = \int_S \mathbf{h}\, dS,$$

while the components of the resultant torque vector $\mathbf{q}_u$ (with respect to the u_{xyz} frame) are given by:

$$\begin{aligned} \mathbf{q}_{u_x} &= \int_S h_z y\, dS, \\ \mathbf{q}_{u_y} &= -\int_S h_z x\, dS, \\ \mathbf{q}_{u_z} &= \int_S (h_y x - h_x y)\, dS. \end{aligned}$$

It should be pointed out that the above relationships for the moment components are only valid for cases when the the contact area warping is negligible; otherwise, contributions of friction components to rolling resistance moments $\mathbf{q}_{u_x}$ and $\mathbf{q}_{u_y}$ should be accounted for. It is sometimes useful to consider also discrete contact points, through which a force of finite value is transmitted. In this case, obvious modifications apply to the integrals above.

According to the Hertz theory of contact mechanics (which is restricted to frictionless, smooth surfaces and perfectly elastic solids), the projection of the contact area S on the osculating plane is an ellipse, symmetric with respect to u. In the following, however, different hypotheses will be assumed. Friction forces will not be neglected; on the contrary, emphasis will be put on their measurement and control, which are fundamental in manipulation operations. No hypotheses will be made about the pressure distribution over the contact area, nor the contact area itself is supposed to have any simple shape or even to be simply connected. This means that contacts occurring on several small areas and/or points (as is the case when irregular surfaces are brought into contact) will be allowed, as long as contact points are close enough. In order to quantify this "close enough", let the *contact convex hull* be defined as the smallest convex portion of the contacting surfaces that encloses every contact point and/or area. Thus, we formulate the only two hypotheses about contact that will be assumed in the following:

Hyp.1: *contact pressures can only be directed towards the inside of the bodies;*

Hyp.2: *the dimensions of the* contact convex hull *are small compared to the radii of curvature of the surfaces in the vicinity of the contact zone.*

The first assumption disregards adhesive forces through contact. This is largely justified for almost every practical case.

The second hypothesis allows one to treat uniformly simple and multiple point contacts, and some cases of contact between compliant or conformal surfaces. For instance, the algorithms proposed in the following can be applied to contacts between finite portions of planes. Equivalent to the hyp.2 is the assumption that the contact convex hull approximately lies on a plane (the *contact plane*, π). It can be noted that the class of contacts considered is much wider than the point-contact-with-friction model assumed in previous literature. The contact convex hull H is represented in figure 2 along with some small areas S_i where contact actually occurs.

Having dropped the Hertzian assumption about the symmetry of the contact area, the choice of u (the point where contact first occurred) as the origin of the reference frame is somewhat abstract and no longer convenient. On the other hand, it can be observed that in the original proposition of this contact type [3, 7], *soft fingers* have been assumed to transmit a force and a moment about the normal axis, without specifying where in the contact area the force is applied and the normal axis is defined. Therefore, the definition of a suitable *reference frame* to characterize soft finger contacts appears to be necessary.

An expedient description of soft finger contact interactions can be obtained as follows. It is well known that any set of forces (in particular the set of contact forces exerted on each body) which is equivalent to a resultant force $\mathbf{p}$ applied at u and a resultant moment $\mathbf{m}_u$, is also equivalent to $\mathbf{p}$ applied to the wrench axis of the set plus a moment $\mathbf{q}_w$ parallel to the same axis. It can be easily demonstrated that, in general, given any arbitrary direction l, there exists a line such that $\mathbf{p}$ applied at points on that line, and a moment $\mathbf{q}_l$ parallel to the given direction, form an equivalent set of forces. Using this result we define the new origin of the soft finger contact reference frame as:

Definition:*A* contact centroid, $\mathbf{c}$, *is a point of a contacting surface such that the set of contact forces is equivalent to a resultant force* $\mathbf{p}$ *applied at* $\mathbf{c}$ *plus a moment* $\mathbf{q}$ *normal to the surface at* $\mathbf{c}$.

The contact centroid has an important property: *if hyp.1 and hyp.2 hold, the contact centroid lies in the contact convex hull.* This can be easily seen if the contact area is flat, since in this case the contact centroid coincides with the centroid of the distribution over S of the normal pressures h_z, which are assumed to be compressive everywhere. A similar property holds even in less strict hypotheses, allowing finite warping of the contact area, but the proof will be omitted here.

This characteristic of the contact centroid is very meaningful for contact sensing. In fact, if the position of the contact centroid can be measured, and an estimate δ of the dimension of the contact area can be reasonably assumed (based e.g on the compliance of the finger material and on the the intensity of contact force), then we are assured

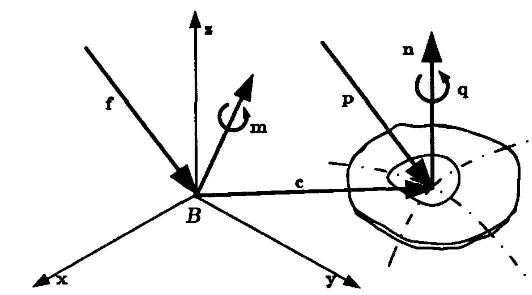

Figure 3: Knowns (force and torque measurement vectors **f** and **m**) and unknowns (contact centroid location **c**, contact force **p** and spin torque **q**) of the problem.

that the point of actual contact furthest from the contact centroid is at a distance less or equal to δ.

The soft finger reference frame will be defined as the *orthonormal* frame c_{xyz} centered in $\mathbf{c}$, whose c_z axis is parallel to the inward normal direction to the contacting surface in $\mathbf{c}$, and whose c_y axis is perpendicular to the resultant force $\mathbf{p}$. The c_{xy} plane will be referred to as the *contact plane* π. In this reference frame, the set of contact forces can be described by their resultant $\mathbf{p}_c = (p_{c_x}\, 0\, p_{c_z})^T$ and by a torque $\mathbf{q}_c = (0\, 0\, q_{c_z})^T$. The p_{c_z} and p_{c_x} components of $\mathbf{p}_c$ are the normal and friction components of the contact force respectively. The only non-null component of $\mathbf{q}$, q_z, is the *spin* resisting torque. The components of $\mathbf{p}$ and q_z, along with the location of the contact centroid on the sensor surface, form a set of data which are sufficient to completely describe the global characteristics of a soft finger contact. The following sections will discuss how these quantities can be elicited from remote force/torque measurements.

3 Problem Formulation

An intrinsic contact sensor consists of a six-axis force/torque sensor and of a finger, whose surface is known. Let B_{xyz} be the reference frame attached to the sensor, and $\mathbf{f}$ and $\mathbf{m}$ the measured resultant force and moment in that frame, respectively. The choice of B_{xyz} is arbitrary, since simple relationships allow to express $\mathbf{f}$ and $\mathbf{m}$ in any other frame rigidly fixed to B_{xyz}. The surface of the fingertip can be described by an implicit relationship as:

$$S(\mathbf{r}) = 0,$$

where $\mathbf{r}$ describes the generic point of 3-D space with respect to B. The surface $S(\mathbf{r}) = 0$ is supposed to have continuous first derivatives everywhere, so that a normal unit vector can be defined at every point on S as:

$$\mathbf{n} = \frac{\nabla S(\mathbf{r})}{||\nabla S(\mathbf{r})||},$$

where ∇ is the gradient operator.

The knowns and unknowns of the intrinsic contact sensing problem are summarized in figure 3. Force and moment balance equations in the reference frame B_{xyz} are written, in vector notation, as:

$$\mathbf{f} = \mathbf{p}, \qquad (1)$$

$$\mathbf{m} = \mathbf{q} + \mathbf{c} \times \mathbf{p}, \qquad (2)$$

where every vector is expressed in the B_{xyz} frame, and the relative subscript has been omitted. Note that the contact force and torque in the sensor frame and in the above-described soft-finger reference frame are easily related as:

$$p_{c_z} = \mathbf{p}^T\mathbf{n}\,; \qquad p_{c_x} = \sqrt{\mathbf{p}^T(\mathbf{I} - \mathbf{n}\mathbf{n}^T)^2\mathbf{p}}\,; \qquad q_{c_z} = \mathbf{q}^T\mathbf{n}\,.$$

To force the contact centroid to lie on the fingertip surface, we impose that:

$$S(\mathbf{c}) = 0. \qquad (3)$$

Finally, since $\mathbf{q}$ is parallel to $\mathbf{n}$, it holds:

$$\mathbf{n} \propto \mathbf{q} = \frac{K}{2}\nabla S(\mathbf{c}), \qquad (4)$$

where K is an unknown constant introduced for convenience.

Expanding equations 1 through 4 yields a non-linear system of ten equations in ten scalar unkowns, i.e. the nine components of $\mathbf{f}$, $\mathbf{m}$, $\mathbf{r}$ plus K. However, by simply substituting equation 1 and 4 in equation 2, the problem is reduced to four equations in four unknowns. Although the equations-unknowns balance is satisfied, the solution of the non-linear system of equations above is not trivial in general. The existence of a solution is not guaranteed for arbitrary $\mathbf{f}$, $\mathbf{m}$ and surface geometries. The uniqueness of the possible solutions cannot be assured unless further assumptions are made about the finger surface (this point will be discussed later). Moreover, a closed-form algorithm to solve the above stated problem is not expected to be available for all but the simplest surfaces. In the following, two methods for solving the above equations are discussed.

4 A Solution For Hard Fingers

In his seminal work [1], Salisbury proposed a method for contact sensing which amounts to a solution of equations 1 through 4 under tighter assumptions than we specified above. In particular, it is assumed that the resistance of friction to spinning relative motion is negligible, i.e. that $\mathbf{q} = 0$ (*hard finger* model of contact). In this case, it can be easily verified that the set of contact forces is equivalent to a pure force applied on the wrench axis. The wrench axis is described in the sensor frame B_{xyz} by the vectorial equation:

$$\mathbf{r} = \mathbf{r}_0 + \lambda\mathbf{f} \qquad (5)$$

where

$$\mathbf{r}_0 = \frac{\mathbf{f} \times \mathbf{m}}{||\mathbf{f}||^2} \qquad (6)$$

The wrench axis is a line through $\mathbf{r}_0$ and parallel to $\mathbf{f}$, parametrized by λ. The contact centroid will be found at the intersections of this line with the sensor surface, so that the problem is reduced to bare geometry, i.e. to solving the scalar equation $S(\lambda) = 0$ in the only scalar unknown λ. *In order to guarantee the uniqueness of the solution*, one further hypothesis of convexity of the fingertip surface must be assumed. In fact, a line intersects a convex surface in at most two points, between which the contact centroid can be easily discriminated by enforcing the criterion $\mathbf{f}^T\mathbf{n}(\mathbf{c}) < 0$, which guarantees the resultant contact force to point inwardly at the sensor surface.

If this solving method (which will be called the *wrench* method) is applied to the more general problem of soft finger contact, where $\mathbf{q}$ is not null, the point so obtained will not coincide with the contact centroid; henceforth, the wrench-method point will be denoted by the vector $\mathbf{c}_w$. It should be pointed out that the point $\mathbf{c}_w$ does not share the above discussed property of contact centroids, so that $\mathbf{c}_w$ could lye far away from the actual location of contact points/areas. Yet, the point $\mathbf{c}_w$ retains a valuable meaning in real conditions as an easy-to-compute approximation of the contact centroid. In order to give an estimate of the distance between $\mathbf{c}_w$ and the contact centroid $\mathbf{c}$, let $\mathbf{e}$ represent the difference vector $\mathbf{e} = \mathbf{c}_w - \mathbf{c}$ (see figure 4).

The two sets of forces, $[\mathbf{f}, \mathbf{q}]$ applied at $\mathbf{c}$, and $[\mathbf{f}, \mathbf{q}_w]$ applied at $\mathbf{c}_w$, are both equivalent with the actual set of contact forces, hence they

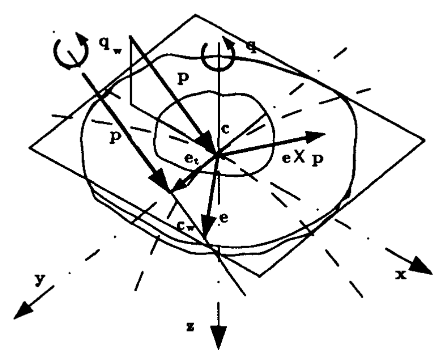

Figure 4: *Approximation of the contact centroid with the wrench-axis method.*

are equivalent to each other. The balance equation of moments about c can be written as:

$$\mathbf{q} = \mathbf{q}_w + \mathbf{e} \times \mathbf{f}. \tag{7}$$

By definition of the wrench axis, $\mathbf{q}_w$ is parallel to $\mathbf{f}$, and is given by

$$\mathbf{q}_w = \frac{\mathbf{f}^T\mathbf{m}}{||\mathbf{f}||^2}\mathbf{f}. \tag{8}$$

By substituting equations 1 and 2 in equation 8, we obtain:

$$\mathbf{q}_w = \frac{\mathbf{p}^T\mathbf{q} + \mathbf{p}^T(\mathbf{c}\times\mathbf{p})}{||\mathbf{p}||^2}\mathbf{p} = \frac{||\mathbf{q}||}{||\mathbf{p}||}\cos\phi\ \mathbf{p}, \tag{9}$$

where ϕ is the angle between $\mathbf{p}$ and $\mathbf{n}$, or *friction angle.* The error vector $\mathbf{e}$ will in general have a component in the contact plane $\mathbf{e}_t$ and a component parallel to the contact force $\mathbf{p}$, $\mathbf{e}_p$. Because of equation 7, $\mathbf{e}\times\mathbf{p}$ is a linear combination of $\mathbf{n}$ and $\mathbf{p}$, hence it will lie on the plane through them, i.e. c_{xz}. Since $\mathbf{e}_t$ is normal to $\mathbf{e}\times\mathbf{p}$, $\mathbf{e}_t$ lies along the axis c_y of the soft-contact frame, and is perpendicular to $\mathbf{e}_p$. Therefore, the length of $\mathbf{e}$ can be given a lower bound if $\mathbf{e}_t$ is known. From equation 7, we have:

$$||\mathbf{e}_t|| = \frac{||\mathbf{q}-\mathbf{q}_w||}{||\mathbf{f}||}.$$

The length of $\mathbf{q}-\mathbf{q}_w$ can be obtained from equation 9 as:

$$||\mathbf{q}-\mathbf{q}_w|| = ||\mathbf{q} - ||\mathbf{q}||\ \cos\phi\,\frac{\mathbf{p}}{||\mathbf{p}||}|| = ||\mathbf{q}||\sin\phi,$$

and hence:

$$||\mathbf{e}|| \geq ||\mathbf{e}_t|| = \frac{||\mathbf{q}||\ ||\mathbf{p}_t||}{||\mathbf{p}||^2}. \tag{10}$$

where $\mathbf{p}_t$ is the friction component of $\mathbf{p}$. In view of this result, it can be observed that the distance between the point found by the wrench method and the contact centroid grows quickly as the effects of friction increase. Then, the approximation of the contact centroid with point $\mathbf{c}_w$ should be avoided if high-friction and/or compliant materials are employed in building the fingertips. Numerical examples of such errors are provided *in the final section of this paper.*

5 An Exact Solution for Soft Ellipsoidal Fingertips

As discussed above, the wrench axis method does *not* provide information about the spin torque exerted by friction on soft fingers. Moreover, this method gives approximations of the contact centroid which are unsatisfactory if high-friction and compliant materials are employed to build the finger. In this section, a different solution to the intrinsic contact sensing problem is proposed, which avoids those shortcomings. In order to simplify the mathematics and to provide a closed-form algorithm, the fingertip surface will be constrained to belong to a specified class of surfaces, namely, to be a quadratic form of the type:

$$S(\mathbf{r}) = \mathbf{r}^T\mathbf{A}^T\mathbf{A}\mathbf{r} - R^2 = 0, \tag{11}$$

where $\mathbf{A}$ is a constant coefficient matrix, and R is a scale factor used for convenience. Since the force/torque sensor reference frame B_{xyz} can be moved arbitrarily in the space, we assume that $\mathbf{A}$ can be written in diagonal form as:

$$A = \begin{pmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{pmatrix}.$$

In order to guarantee the uniqueness of the solutions that will be found, the surface specification must be restricted to convex portions of such a quadratic form; for instance, only one of the sheets of a double-sheet hyperboloid would be an appropriate sensor surface. Such caution is not necessary if only ellipsoids are considered: in such a case, the half-length of the ellipsoid axes are given by αR, βR and γR, with $0 \leq 1/\alpha \leq 1$, $0 \leq 1/\beta \leq 1$, $0 \leq 1/\gamma \leq 1$.

The choice of this class of surfaces is justified by several reasons:

- Quadratic forms can locally approximate, up to the second order, any continuous surface;
- Some very interesting surfaces (e.g. spheres, cylinders, and planes) can be regarded as particular cases of ellipsoids;
- This assumption is standard in contact mechanics (e.g. Hertzian theory of elastic contact), so that formulas providing estimates of the dimensions of the actual contact area (see section 2) can be easily applied.

Substituting equation 11 into equation 4 leads to:

$$\mathbf{n} = \frac{\mathbf{A}^2\mathbf{c}}{||\mathbf{A}^2\mathbf{c}||} \propto \mathbf{q} = K\mathbf{A}^2\mathbf{c}, \tag{12}$$

and substituting this and equation 1 in equation 2, we obtain:

$$\mathbf{m} = K\mathbf{A}^2\mathbf{r} + \mathbf{c}\times\mathbf{f}. \tag{13}$$

Equation 11 and equation 13 form a system of four non-linear equations and four unknowns which can be rewritten in the following form:

$$\begin{aligned} \mathbf{\Gamma}\mathbf{c} &= \mathbf{m} && (14) \\ \mathbf{c}^T\mathbf{A}^2\mathbf{c} &= R^2, && (15) \end{aligned}$$

where $\mathbf{\Gamma} = \mathbf{\Gamma}(K)$ is a 3×3 matrix whose elements are functions of K and of the measured force components f_1, f_2 and f_3:

$$\mathbf{\Gamma}(K) = \begin{pmatrix} K/\alpha^2 & f_3 & -f_2 \\ -f_3 & K/\beta^2 & f_1 \\ f_2 & -f_1 & K/\gamma^2 \end{pmatrix}.$$

The determinant of $\mathbf{\Gamma}(K)$ is given by:

$$\det\mathbf{\Gamma}(K) = K(K^2D^2 + ||\mathbf{A}F||^2),$$

where $D = \det\mathbf{A}$. The matrix $\mathbf{\Gamma}(K)$ is singular for $K=0$, i.e. when the local torque $\mathbf{q}$ is null. In this case, as we showed before, the contact centroid position can be exactly determined by the wrench method. The value of the parameter λ in equation 5 corresponding to the intersection of the wrench axis with the ellipsoid surface is given by:

$$\lambda = \frac{-\mathbf{f}'^T\mathbf{r}'_0 - \sqrt{(\mathbf{f}'^T\mathbf{r}'_0)^2 - ||\mathbf{f}'||^2(||\mathbf{r}'_0||^2 - R^2)}}{||\mathbf{f}'||^2},$$

where $\mathbf{f}' = \mathbf{A}\mathbf{f}$ and $\mathbf{r}'_0 = \mathbf{A}\mathbf{r}_0$ (recall the definition of $\mathbf{r}_0$ in equation 6).

The singularity of $\mathbf{\Gamma}(K)$ can be checked out directly from force-torque measurements by means of the simple equivalent relationship $\mathbf{f}^T\mathbf{m} = 0$. Note also that this equation has sero likelyhood of being verified exactly, if real (noisy) measurements are considered. In any other case, K is not null, and $\mathbf{\Gamma}(K)$ has an inverse $\mathbf{\Gamma}^{-1}(K)$ such that, solving equation 14 for $\mathbf{c}$, we obtain:

$$\mathbf{c} = \mathbf{\Gamma}^{-1}\mathbf{m} = \frac{1}{\det \mathbf{\Gamma}}[K^2D^2\mathbf{A}^{-2}\mathbf{m} + K(\mathbf{A}^2\mathbf{f}) \times \mathbf{m} + (\mathbf{f}^T\mathbf{m})\mathbf{f}]. \quad (16)$$

By substituting this into equation 15, a scalar equation in the only unkown K is obtained as:

$$\mathbf{c}^T\mathbf{A}^2\mathbf{c} = R^2 = \quad (17)$$
$$= \frac{K^4D^4\|\mathbf{A}^{-1}\mathbf{m}\|^2 + K^2\|\mathbf{A}(\mathbf{A}^2\mathbf{f}\times\mathbf{m})\|^2 + (\mathbf{f}^T\mathbf{m})^2(\|\mathbf{A}\mathbf{f}\|^2 + 2K^2D^2)}{K^2(K^2D^2 + \|\mathbf{A}\mathbf{f}\|^2)^2}.$$

After some simple if tedious algebra, it can be verified that:

$$\|\mathbf{A}(\mathbf{A}^2\mathbf{f} \times \mathbf{m})\|^2 = D^2[\|\mathbf{A}^{-1}\mathbf{m}\|^2\|\mathbf{A}\mathbf{f}\|^2 - (\mathbf{f}^T\mathbf{m})^2],$$

so that equation 17 can be simplified in a biquadratic equation as:

$$K^4D^2R^2 + K^2[R^2\|\mathbf{A}\mathbf{f}\|^2 - D^2\|\mathbf{A}^{-1}\mathbf{m}\|^2] - (\mathbf{f}^T\mathbf{m})^2 = 0.$$

Only one of the four possible K solving this equation is real and consistent with the hypothesis of non adhesive contact, and is given by:

$$K = \frac{-\mathrm{sign}(\mathbf{f}^T\mathbf{m})}{\sqrt{2}RD}\sqrt{\sigma + \sqrt{\sigma^2 + 4D^2R^2(\mathbf{f}^T\mathbf{m})^2}}, \quad (18)$$

where:

$$\sigma = D^2\|\mathbf{A}^{-1}\mathbf{m}\|^2 - R^2\|\mathbf{A}\mathbf{f}\|^2,$$

$$\mathrm{sign}(x) = \begin{cases} -1, & \text{for } x < 0 \\ 0, & \text{for } x = 0 \\ 1, & \text{for } x > 0 \end{cases}$$

By substituting back equation 18 in equation 16 and equation 12 gives the complete solution for $\mathbf{c}$ and $\mathbf{q}$, respectively.

5.1 Particular and Limit Cases

Equations 16 and 18 have simpler forms in some particular cases of practical importance:

5.1.1 Sphere

For a spherical sensor surface of radius R centered at the origin of the force/torque reference frame B_{xyz}, the $\mathbf{A}$ matrix equals the identity matrix $\mathbf{I}_3$, and $D = 1$. Hence we have:

$$K = \frac{-\mathrm{sign}(\mathbf{f}^T\mathbf{m})}{\sqrt{2}R}\sqrt{\sigma' + \sqrt{\sigma'^2 + 4R^2(\mathbf{f}^T\mathbf{m})^2}}.$$

where $\sigma' = \|\mathbf{m}\|^2 - R^2\|\mathbf{f}\|^2$.

The contact centroid position on the sphere, for K nonzero, is given by

$$\mathbf{c} = \frac{1}{K(K^2 + \|\mathbf{f}\|^2)}[K^2\mathbf{m} + K\mathbf{f} \times \mathbf{m} + (\mathbf{f}^T\mathbf{m})\mathbf{f}].$$

The solution for the particular case $K = 0$ is given by equation 5 when the value is substituted:

$$\lambda = \pm\sqrt{R^2 - \frac{\|\mathbf{f}\times\mathbf{m}\|^2}{\|\mathbf{f}\|^4}}\frac{1}{\|\mathbf{f}\|}.$$

5.1.2 Cylinder

Consider for example a cylinder having the axis parallel to the B_z axis of the sensor frame, and circular cross section of radius R. Such surface can be described as the limit case of an ellipsoid with characteristic matrix given by:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/\gamma \end{pmatrix}$$

for $\gamma \to \infty$. Applying the same limit to equation 18, we have:

$$K = \frac{-\mathbf{f}^T\mathbf{m}}{\sqrt{R^2\|\mathbf{f}^\perp\|^2 - \|\mathbf{m}''\|^2}},$$

where $\mathbf{f}^\perp = (f_1, f_2, 0)^T$ is the component of $\mathbf{f}$ normal to the cylinder axis, and $\mathbf{m}'' = (0, 0, m_3)^T$ is the componenet of $\mathbf{m}$ parallel to the same axis. If $K = 0$, the wrench method (equation 5) should be applied. Otherwise, the contact centroid on the cylindrical surface of the fingeritp is given by:

$$\mathbf{c} = \frac{1}{K\|\mathbf{f}^\perp\|^2}[K^2\mathbf{m}'' + K\mathbf{f}^\perp \times \mathbf{m} + (\mathbf{f}^T\mathbf{m})\mathbf{f}].$$

5.1.3 Planes

An ellipsoid with matrix $\mathbf{A}$ of the form

$$\mathbf{A} = \begin{pmatrix} 1/\gamma & 0 & 0 \\ 0 & 1/\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

degenerates, for $\gamma \to \infty$, in a couple of parallel planes perpendicular to the B_z axis, at a distance $\pm R$ from B. If $\mathbf{f}'' = (0, 0, f_3)^T$ is the contact force component parallel the B_z axis, equations 18 and 16 become:

$$K = \frac{-\mathbf{f}^T\mathbf{m}}{R\|\mathbf{f}''\|}$$

and

$$\mathbf{c} = \frac{1}{\|\mathbf{f}''\|^2}[\mathbf{f}'' \times \mathbf{m} + R\|\mathbf{f}''\|\mathbf{f}].$$

It is observed that the last formula holds even with $K = 0$.

6 Discussion

One of the main points addressed in this paper is the definition of contact centroid for soft finger contacts, and the derivation of an algorithm for calculating its position starting from remote force/torque measurements. The interest of the contact centroid for characterizing soft fingers contacts follows from its property of being located inside the convex hull that encloses every acual point or area of contact. To illustrate this, a simple numerical example will be worked out. Assume that the real pattern of contact on the surface of a spherical sensor is comprised of only four points $\mathbf{c}_1 - \mathbf{c}_4$, located on top of the sphere as shown in figure 5, such that their projections on the horizontal plane form a rhombus with semi-diagonal δ and 8δ. Let the local contact forces exerted at these points be $\mathbf{h}_1 = (-h_f, h_f, -1)$, $\mathbf{h}_2 = (-h_f, h_f, -1)$, $\mathbf{h}_3 = (h_f, h_f, -1)$, and $\mathbf{h}_4 = (h_f, h_f, -1)$, respectively. Table 1 gives the x-coordinates of the contact centroid $\mathbf{c}$ (calculated through the algorithm proposed in section 5) and of the wrench method point $\mathbf{c}_w$ (section 4) corresponding to different values of δ and h_f. As it can be seen, the two results diverge as the distance δ and the friction force h_f increase. It can be noted that, for $\delta \to 0$, the contact tends to a point-contact-with-friction model, while for small δ the soft-finger model assumptions are verified. Note also that, although for large values of δ the hypothesis 2 of section 2 is not satisfied, the contact centroid retains the characteristic of remaining inside the contact points, while the point obtained by the wrench method do not.

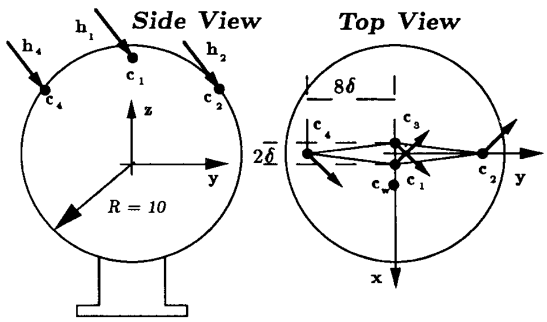

Figure 5: A simple contact pattern used as an example.

δ / h_f	0.125	0.25	0.5
0.1	c : 0.000	c :0.000	c :0.000
	c_w : 0.005	c_w : 0.010	c_w : 0.020
0.7	c : 0.000	c :0.005	c :0.039
	c_w : 0.164	c_w : 0.329	c_w :0.658
1	c : 0.001	c :0.010	c :0.077
	c_w : 0.250	c_w : 0.500	c_w :1.000

Table 1: Position of the contact centroid c and of the approximated (wrench method) point c_w for different values of δ and h_f.

The other important advantage of the proposed algorithm is that the spin resisting torque originated from friction forces is calculated, without adding any new measurement.

Devices based on the force-based contact sensing approach have been actually implemented, and effectively employed in robotic hands. For a discussion on the realization of force/torque sensors on small size robot fingertips, see [4, 2]. The approach followed in the latter paper consists in the application of optimal design techniques to miniaturized force/torque sensors, and is more thoroughly discussed in [8].

The applications of intrinsic contact sensors to robotic manipulation are various, and several have already been experimentally verified. Although it is not possible to detail these applications here, they will be cited for reference:

- The exploration of unknown objects by probing with an intrinsic tactile sensor, and the reconstruction of their surface profile has been described by [4], and later by [5]. Both authors employed the wrench method algorithm. In [9] and [10], the exploration has been performed using the more precise soft finger method, and an hybrid control scheme, which allowed continuous control of the normal component of contact force.

- The capability of intrinsic contact sensors to evaluate the friction components of the contact force and the spin torque (which is unique among other available sensing devices), has been used to measure the coefficients of friction of various objects [9]. This information in turn has been used to discriminate objects on the basis of their apparent friction, and to plan subsequent slippage-safe operations of the hand.

- A real-time control method for augmenting the stability of the grasp of unmodeled objects against slippage has been discussed and demonstrated (in a rather simple setting) in [11]. The ability of the algorithm described in this paper to provide a measurement of the spin resisting torque is instrumental in detecting and reacting to spin slippage danger, whose importance is often underestimated in manipulation planning and control.

The exploitation of the information provided by sensors such as those described in this paper can be expected to allow improvements in many areas of fine manipulation control, since they provide direct, prompt and reliable feedback of fundamental contact characteristics. For instance, intrinsic contact sensors could be profitably used to improve the accuracy of the control of micro-motions of manipulated objects or tools, especially in the presence of slipping and/or rolling contacts.

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